Impact of centroid error and edge effects on Shack-Hartmann wavefront sensor accuracy

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Outline

• Introduction
  – Shack-Hartmann wavefront sensors
  – Motivation

• Analysis
  – SHWF Algorithms
  – Uncertainty analysis

• Uncertainty measurement
  – Experiment
  – Results

• Edge effects

• Conclusions
Understanding precision and accuracy is critical for most applications

Key questions

• How does the accuracy scale?
• What are the limiting factors for accuracy and precision?
• What components affect performance?
• What is the end-to-end performance of these systems?
• Can the total error be quantified?
• How do edges affect the errors?
• What strategies can be employed to minimize error effects?
There are three steps to the analysis process

1. Locate position of focal spot $x_{ij}$

2. Compute wavefront slope

$$\frac{\hat{x}_j - \hat{x}_{io}}{f} = \left( \frac{\partial \phi_j}{\partial x} \right)_i$$

3. Recover wavefront $\phi_j$ by integrating.

Wavefront Analysis

Photomicrograph of discrete level lens array fabricated in fused silica using binary optics technology; lenslets are 250 μm in diameter.
Wavefront analysis:
Centroid

- Determination of the focal spots is usually obtained through a “centroid” calculation.
- The centroid method sets the basic measurement accuracy and dynamic range.
- Pixels are everything for lenslet accuracy.
- Trade between centroid accuracy, and lenslet fitting error.
- Note: Lenslet Fresnel number held constant for this study ($N_{Fr}=4$)

$$N_{Fr} = \frac{d^2}{f\lambda}$$

$$\bar{x}_k = \frac{\sum_{ij \in AOI_k} S_{ij} x_{ij}}{\sum_{ij \in AOI_k} S_{ij}}$$

- 2 mm (72 μm) 9 pixels/spot
- 8.2 mm (144 μm) 49 pixels/spot
- 25 mm (252 μm) 151 pixels/spot
Wavefront analysis: Gradient calculation

- Wavefront gradients are computed from centroids and reference
- Reference file affects accuracy
  - Averaging
  - Systematic errors
  - Calibration
- Gradients can be used to determine out-of-range conditions, and detect areas where the wavefront changes rapidly
- Fitting error is determined by the lenslet size
Wavefront analysis: Wavefront reconstruction

• Reconstruct wavefront from gradient measurements
  \[ \nabla w = \frac{\partial w}{\partial x} \hat{i} + \frac{\partial w}{\partial y} \hat{j} \]

• Modal and Zonal reconstructors
  – Zonal: Zone by Zone integration
    • Matrix iterative or LSQ
    • High spatial resolution information
    • No fit parameters (focus, astigmatic parameters, etc.)
  – Modal: Fit to modes or polynomials
    • Taylor and Zernike polynomials
    • Smoothing functions eliminate noise
    • High spatial frequency information lost
The SHWFS wavefront is a piecewise planar approximation

- Shack-Hartmann sensors measure average tilt over lenslet aperture
- Focal spot irradiance distribution depends upon total incident wavefront
- Low resolution SHWFS measurements tend to UNDERPREDICT the actual wavefront
- Residual WFE error is the RMS difference between the linear approximation and the actual wavefront
Fitting error leads to focal spot degradation

- **Small lenslets**
  - Sample small portion of the wavefront
  - Sampled wavefront matches well to linear approximation
- **Large lenslets**
  - May have large residual wavefront error
  - Reduced Strehl may affect centroid location

There is a trade between the fitting error and the centroid estimation accuracy
SHWFS Precision and Accuracy

- Precision: Repeatability
  - How repeatably can the sensor measure the same parameters
  - Measured by RMS variations, or the variation in a given parameter (tilt, radius of curvature, etc.)

- Accuracy
  - How accurately can the sensor measure a known parameter
  - May vary as a function of input parameter
  - May depend on correlated effects
  - May be driven by more than just noise
A Shack-Hartmann sensor can have incredible precision

- Static environment (isolated from turbulence)
- 100 measurements for statistics
- WFE average RMS: 0.000527 μm (1/1200\textsuperscript{th} wave at 0.635 μm)
- Definition: Centroid Estimation Error (CEE)
  - CEE Average RMS: 0.048 μm (1/200\textsuperscript{th} pixel)!
- Sources of error:
  - Electronic noise
  - Digitization error
  - Pixelization

\[
\nu^2 = \frac{1}{N} \sum_{n=1}^{N} \left( \frac{1}{K} \sum_{k=1}^{K} (x_k - \bar{x}_k)^2 + (y_k - \bar{y}_k)^2 \right)_n
\]
However, the accuracy is significantly worse than repeatability

- RMS WFE across dynamic range: 0.01 μm max
  - 21 times the precision
  - But still λ/50

- Accuracy must be determined by comparison to known quantities
  - Several different quantities can be easily varied in known way
    - Tilt
    - Focus
    - Coma
  - Phase plates can be used for higher order determination
  - Correlated effects may cause worse inaccuracies for low order terms

Setup for varying tilt in a known manner
Calibration accuracy

- Camera was calibrated with nominal focal length
- Check with average tilt measurement
  - Slope error -0.0023
  - Represents a 18 μm error in focal length (.2%)
  - Revised spacing: 8.208
- Typically once calibrated, the lenslet position is adjusted with shims
- No other calibration factors are relevant for SHWFS
Sensor accuracy depends primarily on Centroid Estimation Accuracy

- Difference in precision and accuracy is attributable to the centroid estimation process
- Pixelization plays the dominant role
- Key parameter pixels/focal spot
- Periodic behavior is evident
  - Period: 1 pixel
Gradient Estimation Accuracy is strongly dependent on the pixels/focal spot

- The gradient error depends upon both reference and data
- Pixelization can be the most important phenomenon
- Reference file may become more important for accurate sensors

\[
\begin{array}{c|cc|c|c}
\text{f: 2.047 mm, d: 0.072 mm} & \text{DVArms X} & \text{DVArms Y} & 8.19 \text{ mm} & 49 \text{ pixels} \\
\text{f: 25.1 mm, d: 0.252 mm} & \text{DVArms X} & \text{DVArms Y} & 25.09 \text{ mm} & 151 \text{ pixels} \\
\end{array}
\]
Reconstruction

- The wavefront error depends upon the gradient (and hence the pixelization) error
- RMS wavefront error also depends upon the reconstructor type
- Correlated errors can influence the reconstruction errors
- The residual error is wavefront dependent
- Correlated errors are usually the worst errors that are encountered
WFS scaling - reconstruction

- $N_{Fr} = 4$
- Pixelization dominates up to 8 mm
- Scaling is not quite log-linear
- Results follow centroid effects
- May be different for different reconstructors
- More data needed
For ophthalmic applications, we always have a pupil

- Pupil is not round
- It may not be correctly imaged
- It crosses the lenslet grid in many different ways
- Potentially affects a large number of lenslets
- Yet we want a measurement all the way to the edge
Pupil edge effects

• Calibration error
  – Lenslet focal length is difficult to determine
  – Spacing can be determined accurately
    • But mechanical tolerances limit separation tolerance
  – Spacing error leads to an error in the centroid estimate
Pupil edge effects

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Pupil edge effects

- Calibration error
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  - Spacing error leads to an error in the centroid estimate
- Centroid shift proportional to spacing error
- Log irradiance shown
A typical edge leads to a number of different shapes.

Since the intensity drops, there is relatively little effect on the centroid position.
Strategies for minimizing the error

- Accurate calibration
  - Determine lenslet focal length
  - Set spacing accurately
- Weight by total irradiance
- Increase resolution
- Fit to higher order
  - Discard highest order terms
Edge errors can affect Zernike fit accuracy

- Full pupil reconstruction
- Small to modest edge errors
- Vary fit order, examine 2\textsuperscript{nd} and 3\textsuperscript{rd} order terms

Use a fit order that is 2 orders higher than the desired term
Strategies for minimizing the error

- **Accurate calibration**
  - Determine lenslet focal length
  - Set spacing accurately
- **Weight by total irradiance**
- **Increase resolution**
- **Fit to higher order**
  - Discard highest order terms
- **Mask partial lenslets**
Is accuracy improved by trimming the edge data?

- 5mm reconstruction
  - Originally 5.9 mm
- All edge lenslet trimmed from data
- Vary fit order, examine 2nd and 3rd order terms

Some improvement in accuracy, but fit order is still important
Conclusions

• Centroid accuracy
  – Limited by pixelization NOT electronic noise
  – Scales with pixels/focal spot
  – Focal spot degrades for larger lenslets

• Edge effects
  – Driven by sensor calibration errors
  – Edge effects can drive Zernike orthogonality errors
  – Can be impacted by resolution

• Minimization strategies
  – Maintain > 35 pixels/focal spot
  – Careful calibration to set $L_H = f$
  – Use higher order fits even for low order parameters
  – Weight by irradiance distribution
  – Use higher resolution if possible
WFS design parameters

- Lenslet Fresnel number:
  \[ N_{Fr} = \frac{d^2}{f \lambda} \]

- Focal spot size:
  \[ \rho = \frac{f \lambda}{d} \]

- Angular dynamic range:
  \[ \rho = \frac{d}{N_{Fr}} \]

- Total wavefront dynamic range:
  \[ \frac{w_{\text{max}}}{\lambda} = \frac{N_I N_{Fr}}{4n} \]
Residual wavefront error depends strongly on resolution

- Linear approximation fit error
- In practice, this fit error reduces the lenslet Strehl ratio
- Strehl ratio reduction can significantly affect the accuracy below 10 lens/pupil
The wide range of spatial frequencies present in the data present a problem for reconstruction.

- Matrix iterative reconstructor response apparently rolls off at high frequency (short wavelengths).
- A chirped sine wave, constant amplitude, corrugated surface was generated as a test case.
- Least squares fit to damped amplitude provides an estimate of the frequency response.