Astigmatic Refraction Using Peaks of Interferogram MTF for a Talbot Moiré Interferometer

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Acknowledge coauthors

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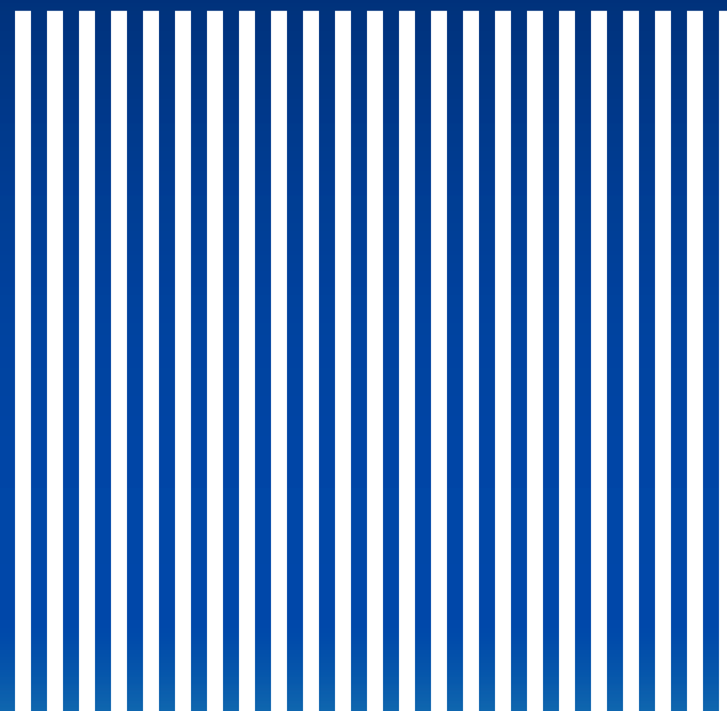
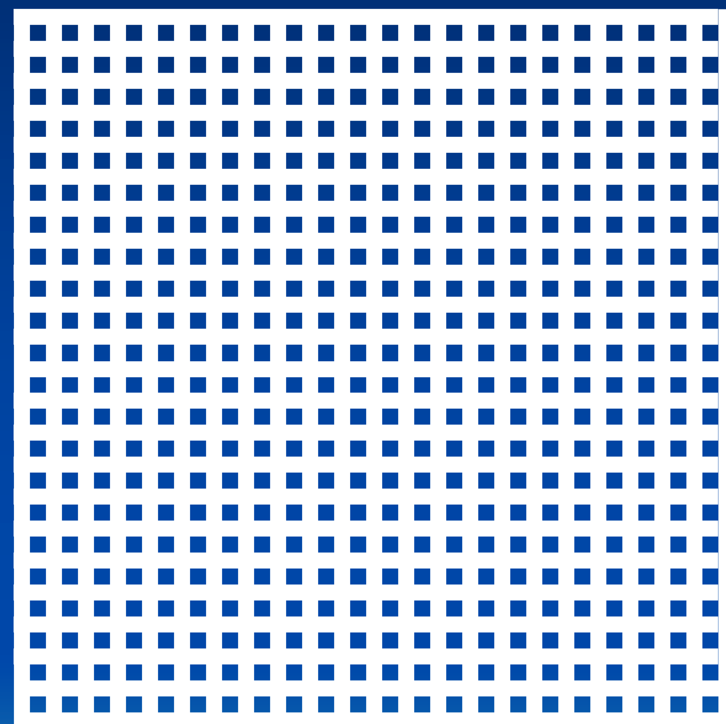
Ronchi Ruling

Set of opaque, parallel bars
with 50% duty cycle

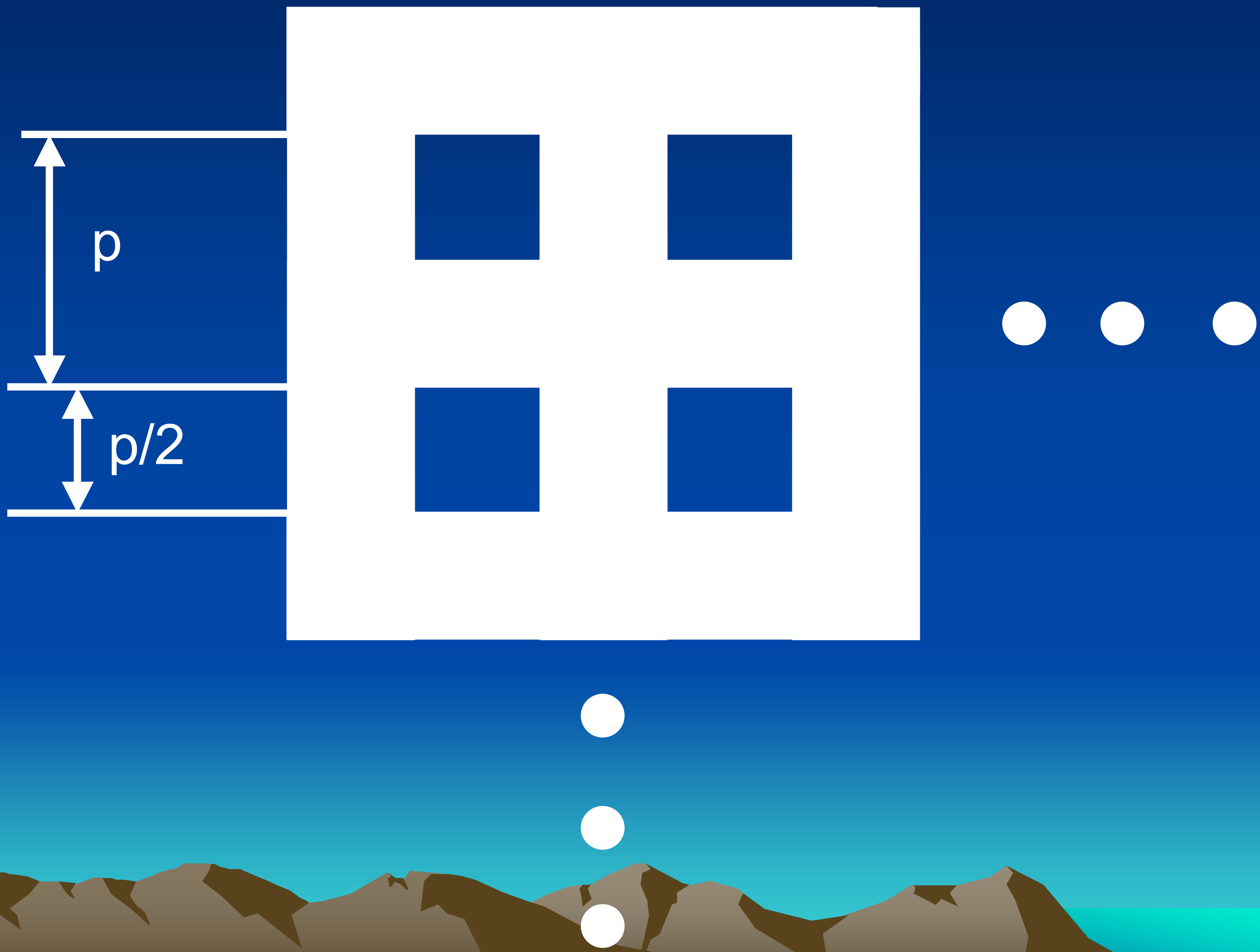


Ronchi Grid

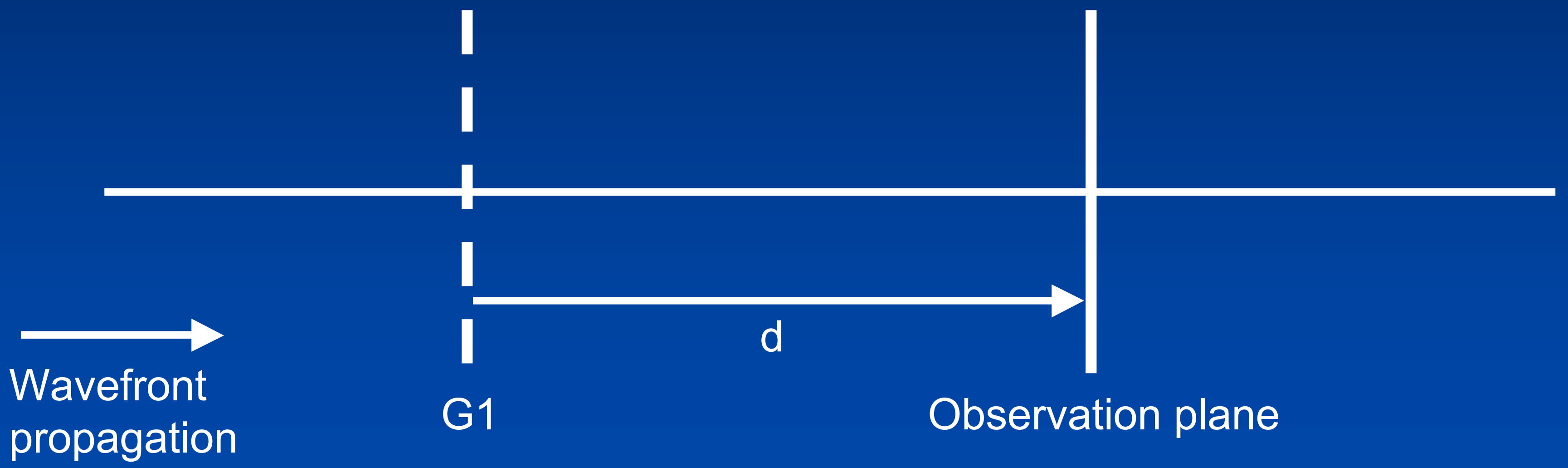
Two Ronchi rulings oriented orthogonal to each other



Ronchi Grid Parameters



Ronchi Grid as Transmission Function



For simplicity, let's model amplitude transmission[1] as a sinusoid…

$$T(x,y)=\frac{1}{2}\left[1+\cos\left(\frac{2\pi}{d}x\right)\right]$$

[1]Goodman, Fourier Optics, 3rd edition, Roberts & Company, p. 88, 2005.

Fresnel diffraction leads to intensity distribution[2] given by…

$$I(x,y)=\frac{1}{4}\left[1+2A\cos\left(\frac{2\pi}{p}x\right)+\cos^2\left(\frac{2\pi}{p}x\right)\right]$$

where

$$A=\cos\left(\frac{\pi\lambda}{p^2}d\right)$$

[2]Goodman, Fourier Optics, 3rd edition, Roberts & Company, p. 89, 2005.

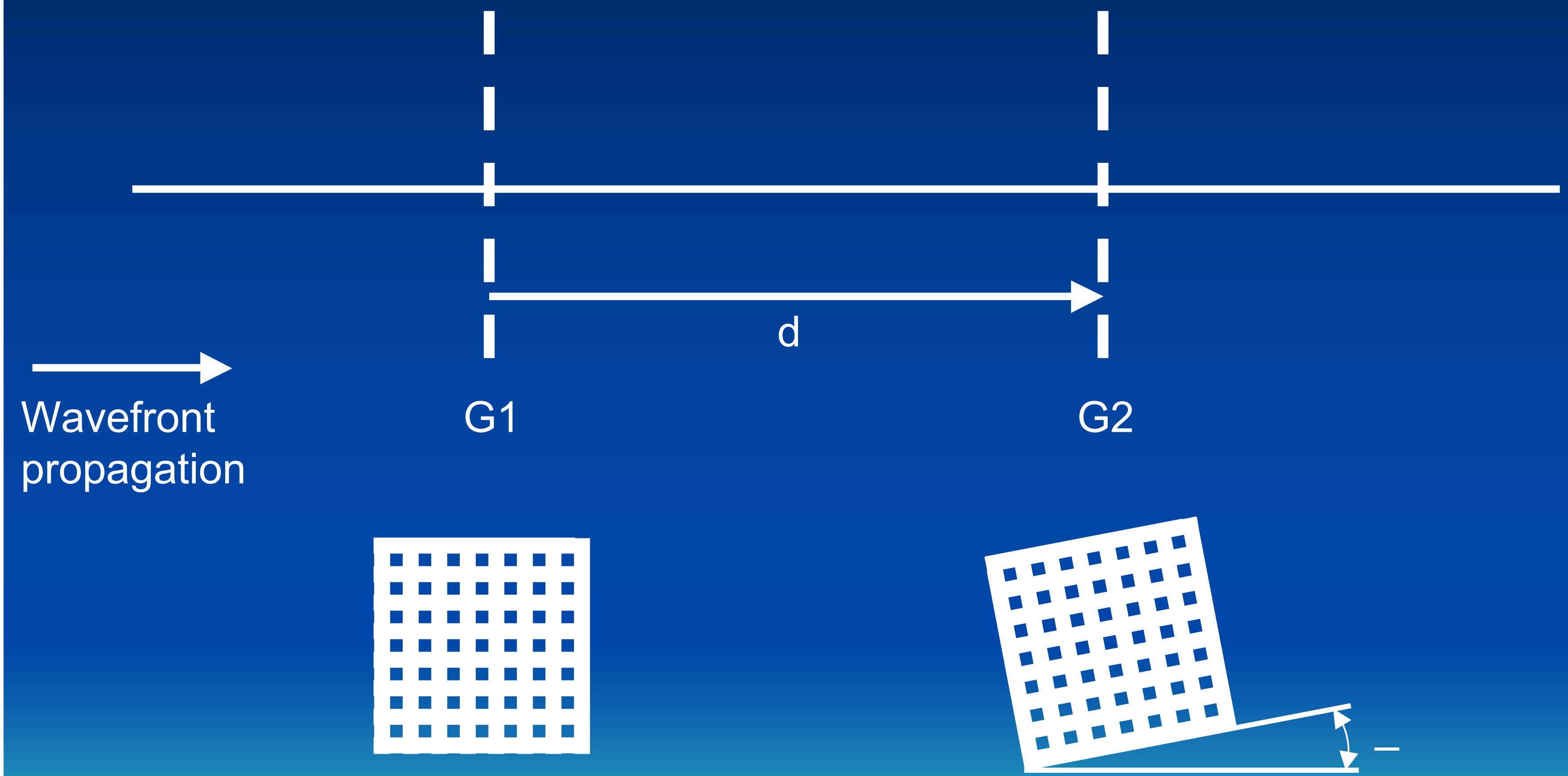
Talbot plane…

$$A = \cos\left(\frac{\pi\lambda}{p^2} d\right)$$

A = ± *1* provides maximum contrast and exact replica of transmission plane. Corresponding *d* location is called ***Talbot plane***.

Now we place a second Ronchi Grid at a Talbot distance d…

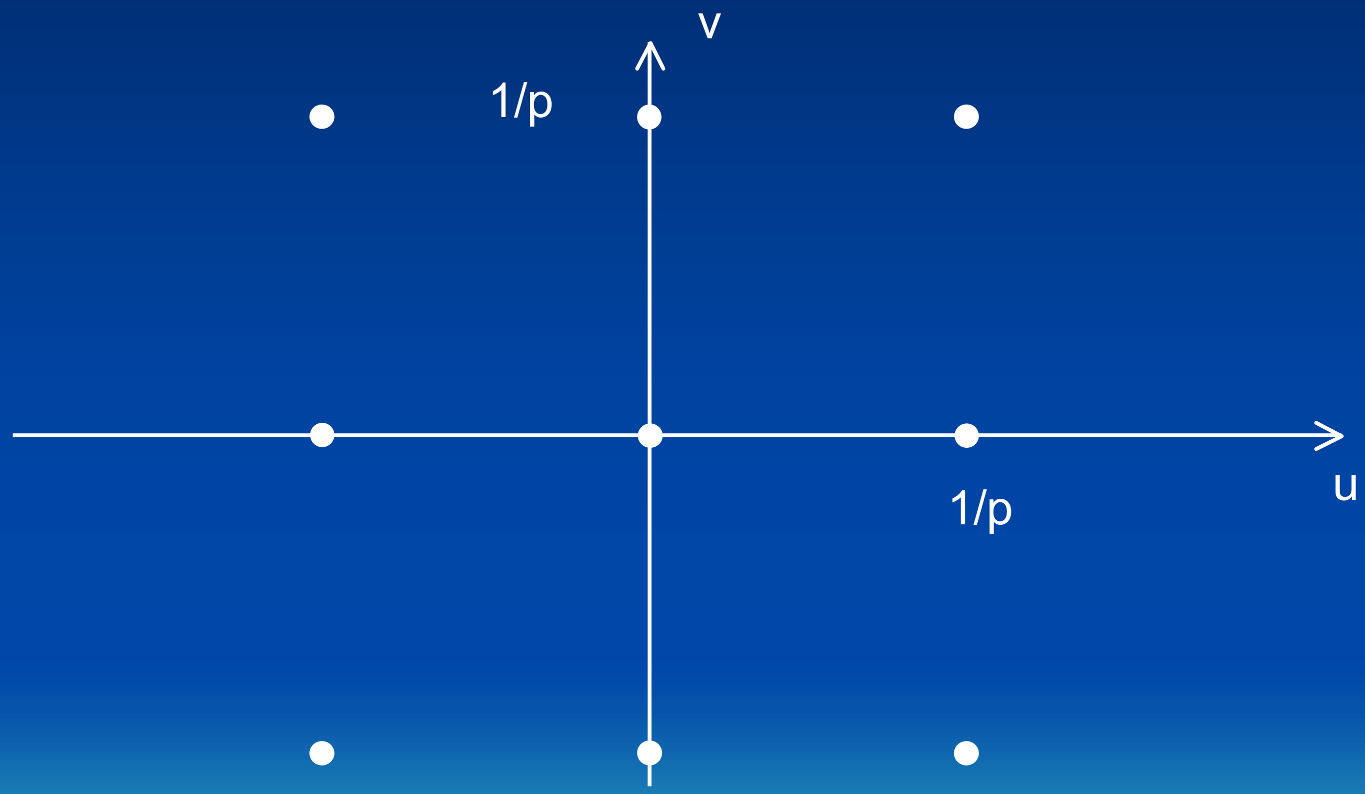
…And rotate the second grid



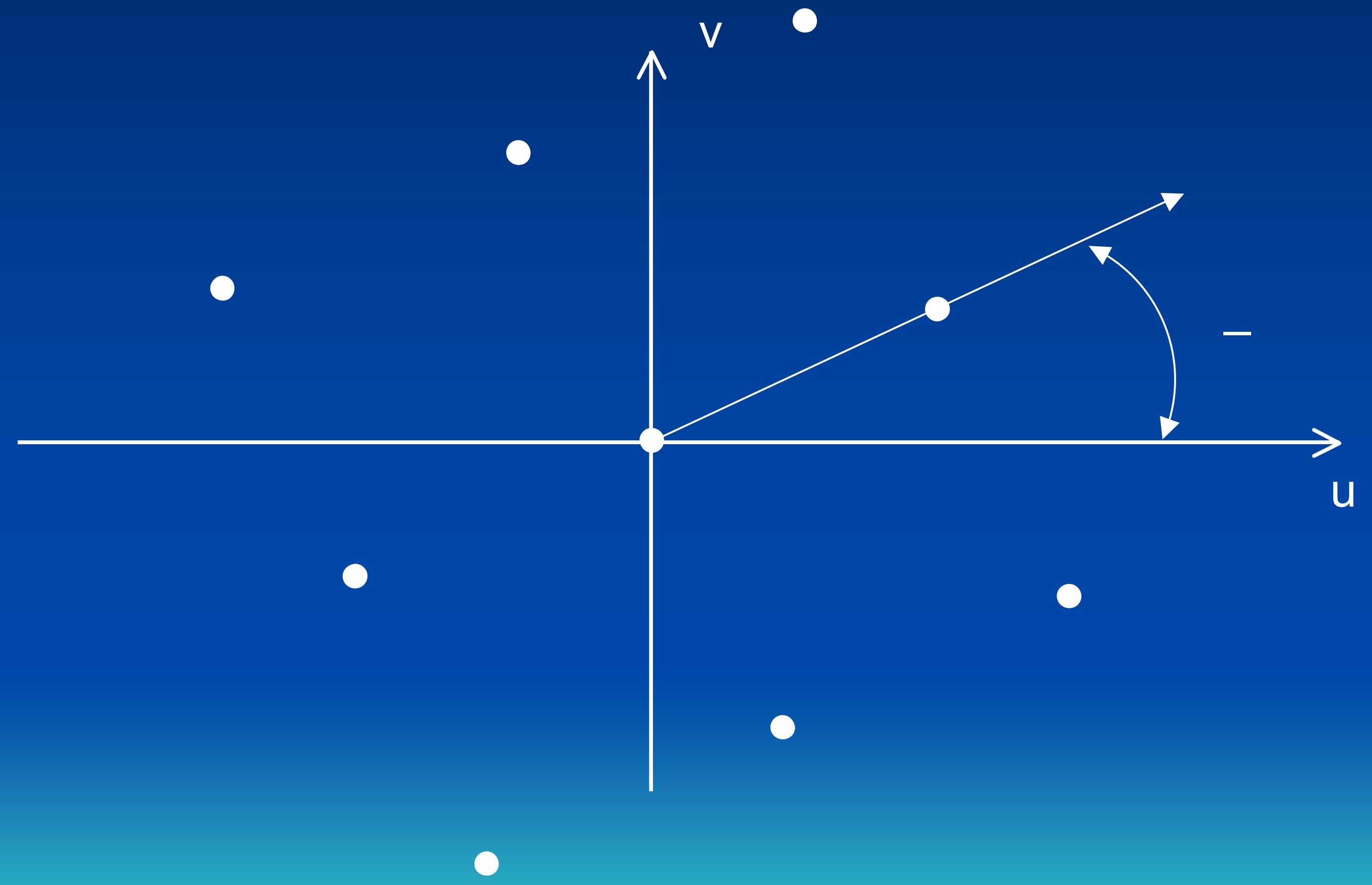
This superposition of grid patterns leads to a moiré pattern. The combination of the moiré effect with the Talbot distance requirement leads to the name…

Talbot Moiré Interferometer

Fourier domain primary peaks of G1 are…



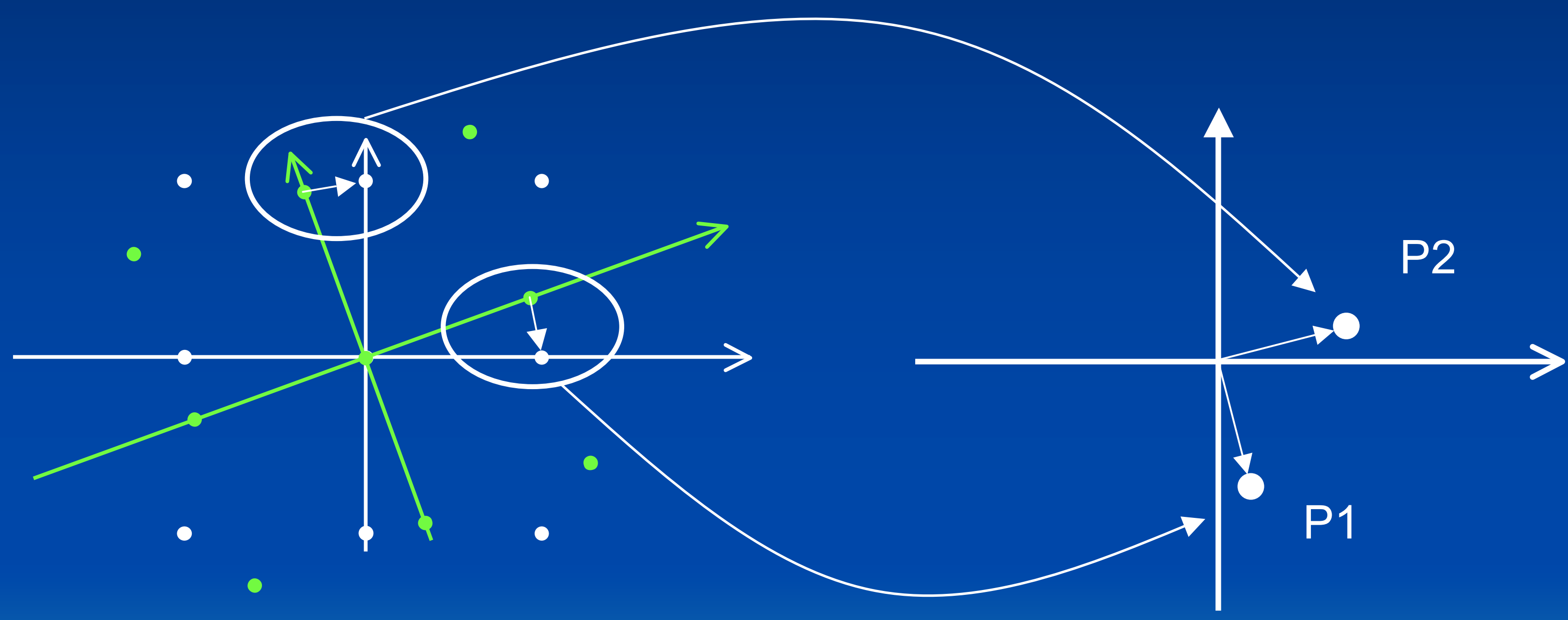
Fourier domain primary peaks of *rotated* G2 are…



In the spatial domain the propagated transmission of G1 is multiplied by the transmission function at G2.

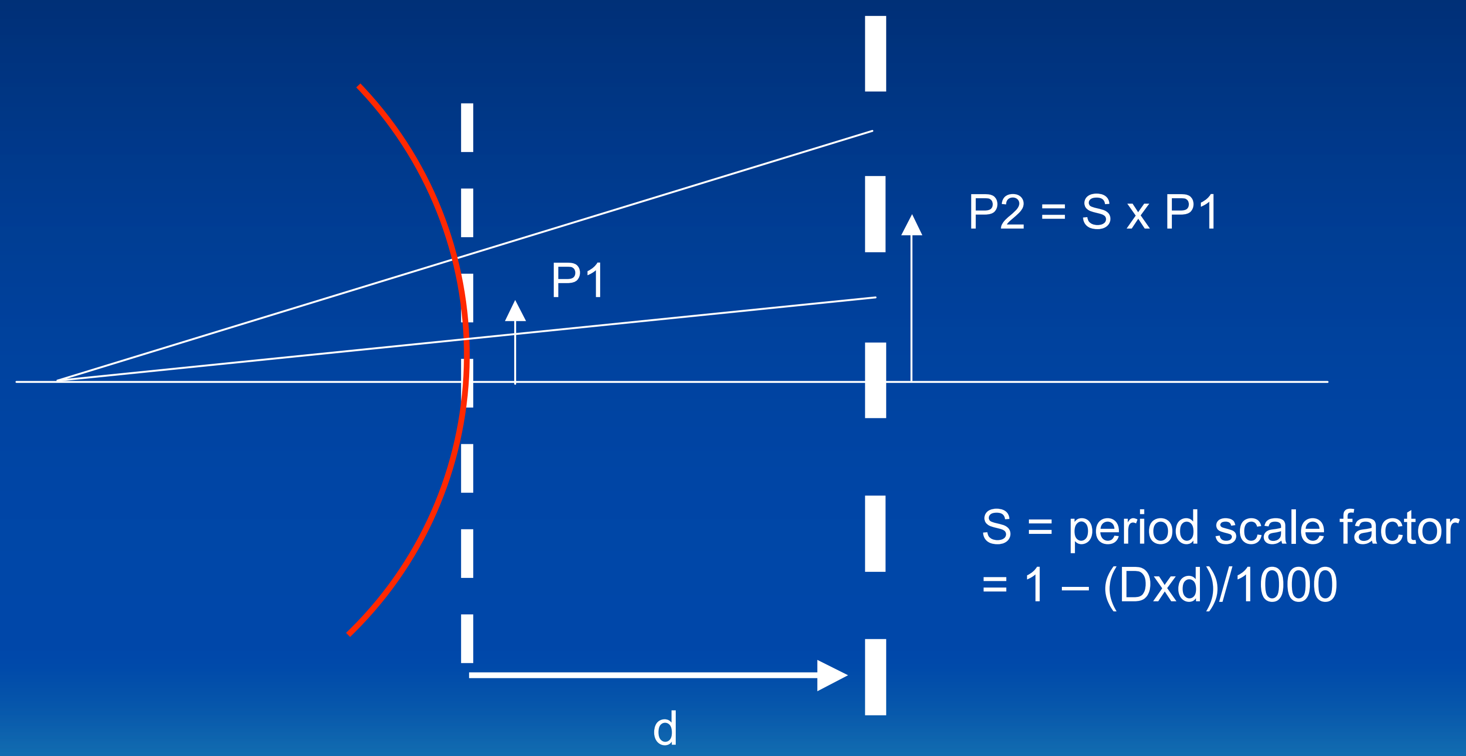
The equivalent operation in the Fourier domain is convolution…

Two primary peaks of MTF of Intensity distribution at G2

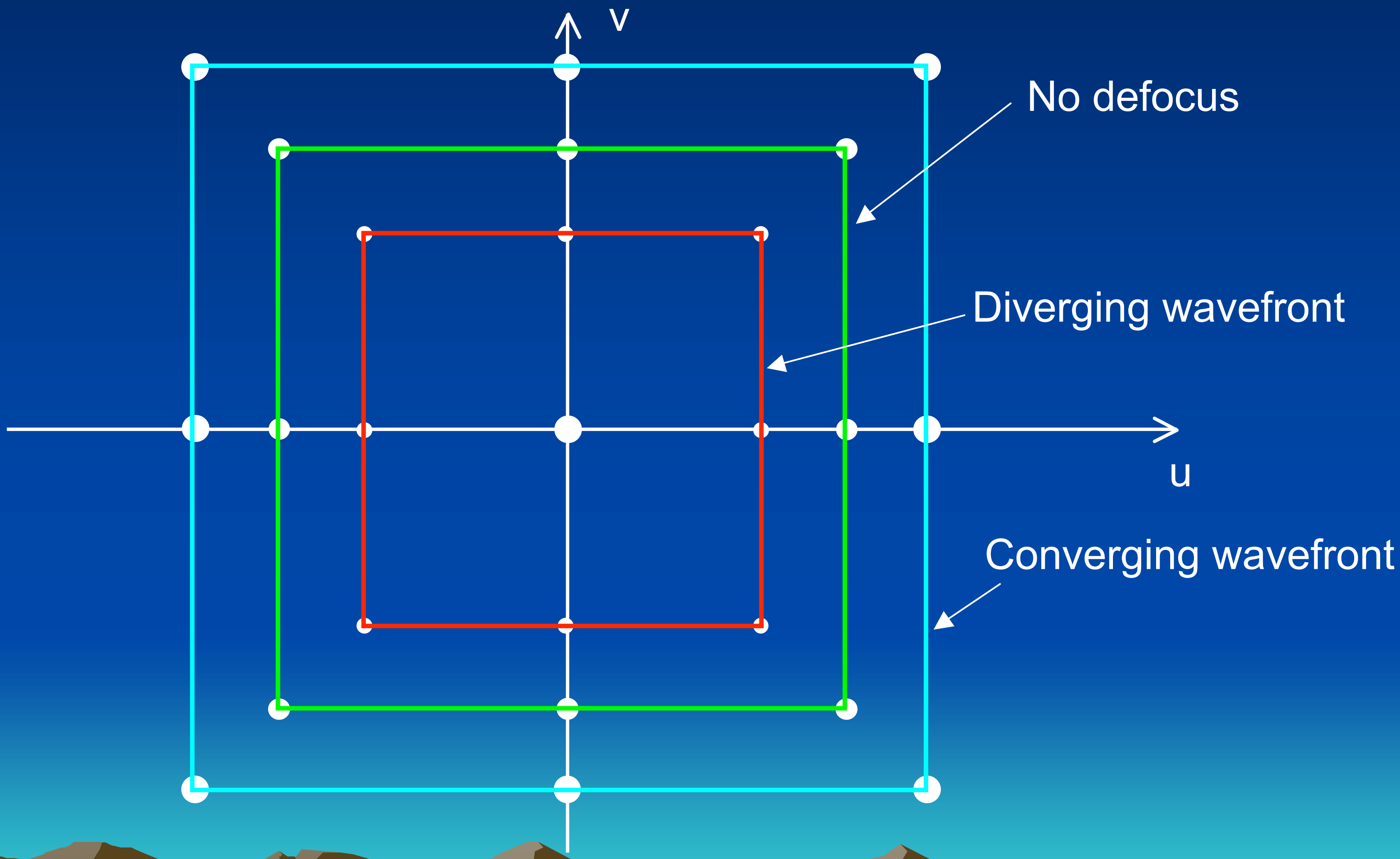


What happens when the incident wavefront at G1 has defocus?

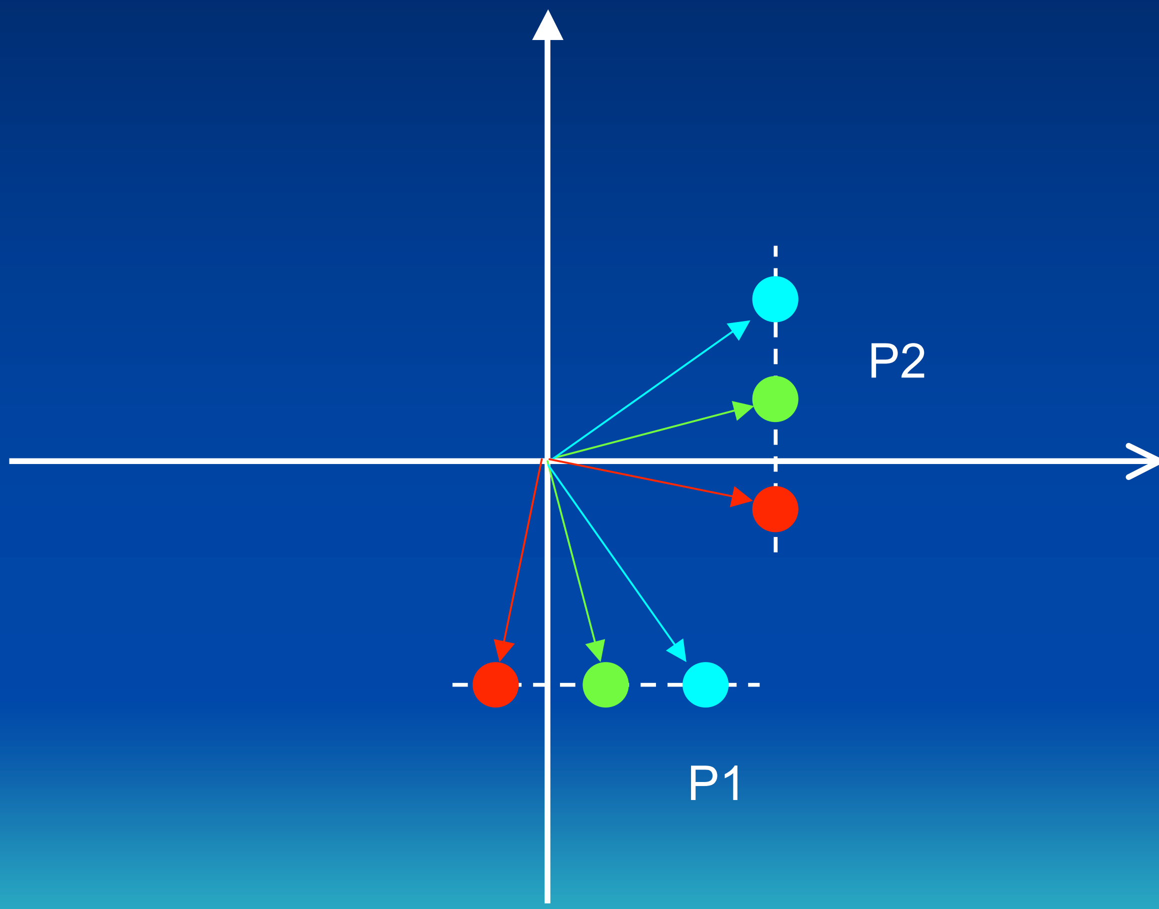
With diverging incident wavefront…



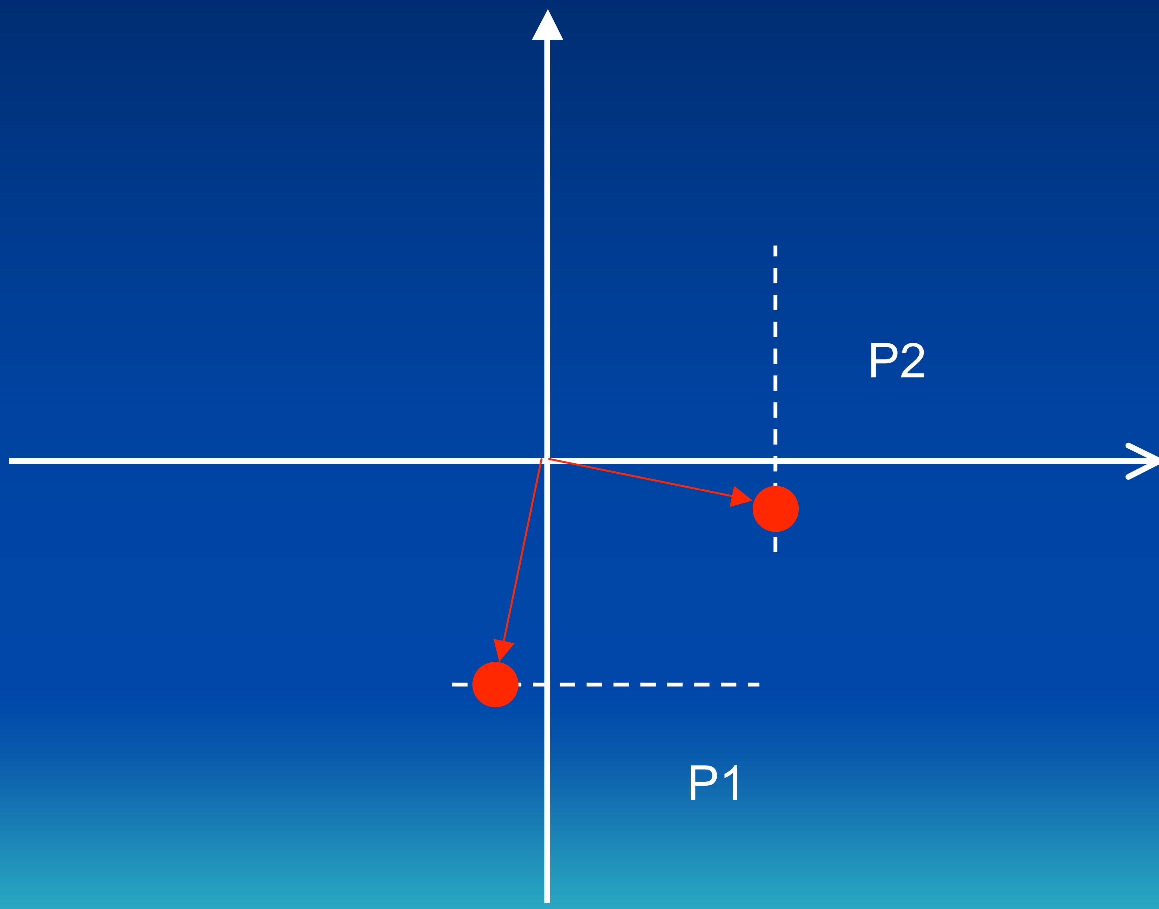
Fourier domain for scaled G1…



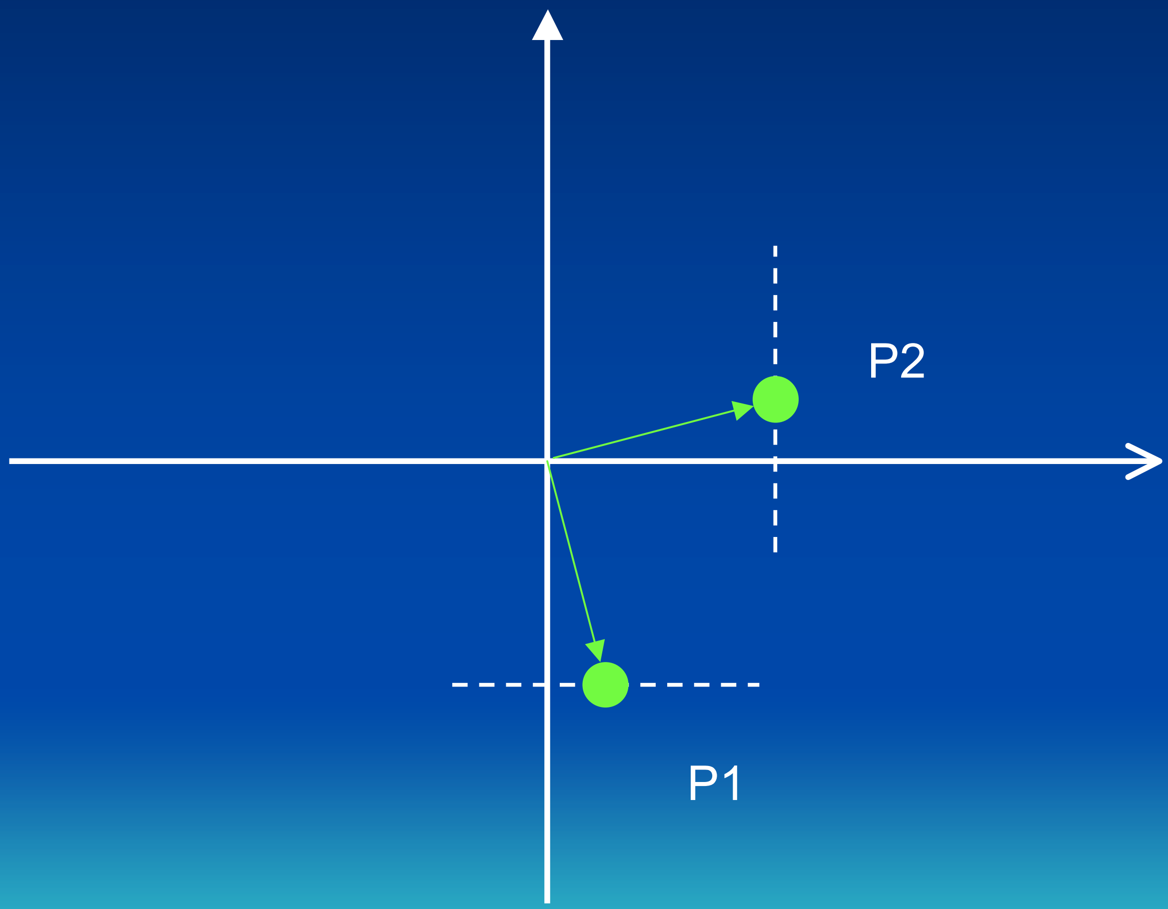
After modulation by G2…



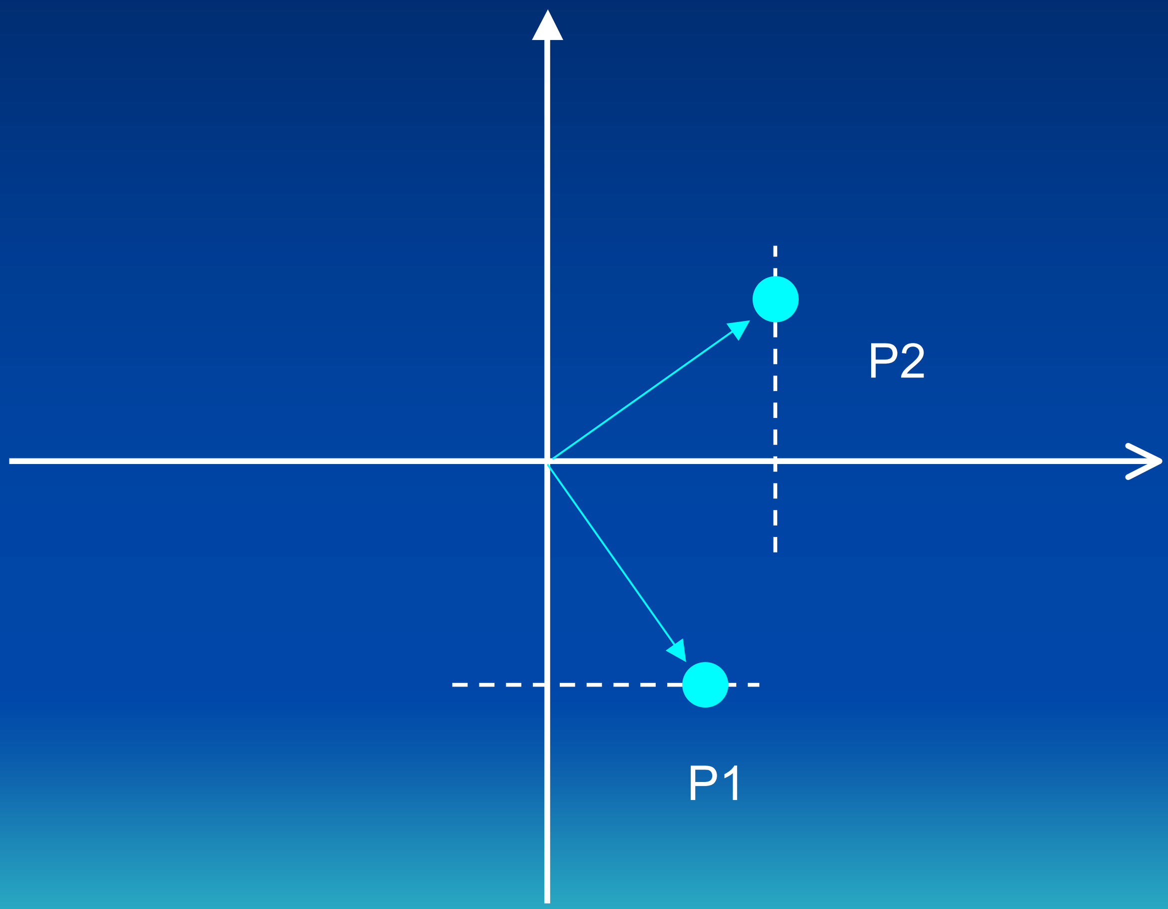
Apparent rotation due to defocus…



Apparent rotation due to defocus..

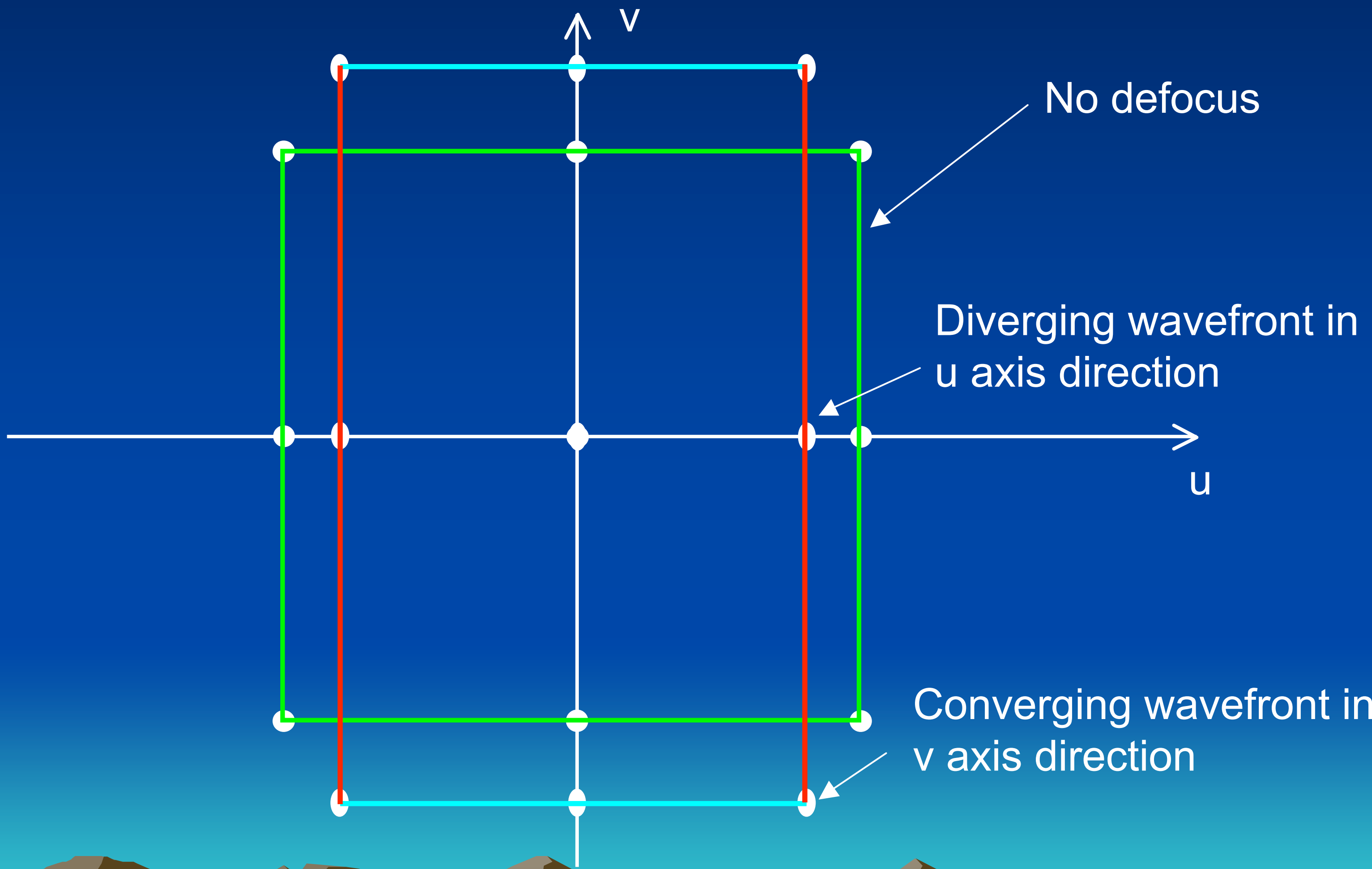


Apparent rotation due to defocus..



Instead of pure defocus, what about astigmatic wavefront?

Astigmatic wavefront aligned with system axes…



General Astigmatic wavefront

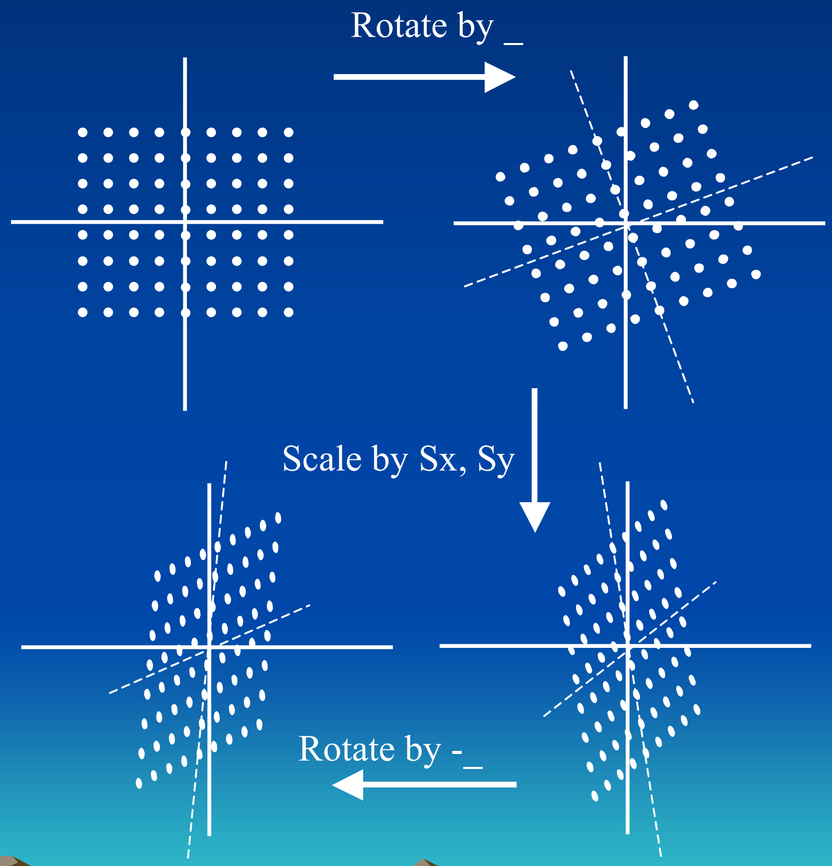
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = c^2 sx + s^2 sy$$

$$B = cs(sy - sx)$$

$$C = s^2 sx + c^2 sy$$



Sx and Sy are related to power in the principal axes.

Given MTF peaks, need to calculate matrix coefficients A, B, and C

Compute Affine Transform…

Since for each G1 point: $R'_N = \begin{bmatrix} A & B \\ B & C \end{bmatrix} R_N$

Solve $$x = \left(A^T A\right)^{-1} \left(A^T b\right)$$

Where…

$$A = \begin{bmatrix} R0_x & R0_y & 0 \\ 0 & R0_x & R0_y \\ R1_x & R1_y & 0 \\ 0 & R1_x & R1_y \\ R2_x & R2_y & 0 \\ 0 & R2_x & R2_y \\ R3_x & R3_y & 0 \\ 0 & R3_x & R3_y \end{bmatrix}, \quad b = \begin{bmatrix} R'0_x \\ R'0_y \\ R'1_x \\ R'1_y \\ R'2_x \\ R'2_y \\ R'3_x \\ R'3_y \end{bmatrix}, \quad x = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

Then, find eigenvalues…

$$\lambda_{1,2} = \frac{(A + C) \pm \sqrt{(A - C)^2 + 4B^2}}{2}$$

Note: ALWAYS two real eigenvalues.

Scale factors and axis…

$$S_{1,2} = \frac{1}{\lambda_{1,2}}$$

$$Axis = \tan^{-1}\left(\frac{\lambda_1 - A}{B}\right)$$

Note: Use atan2 function to handle zero valued B.

Principal powers…

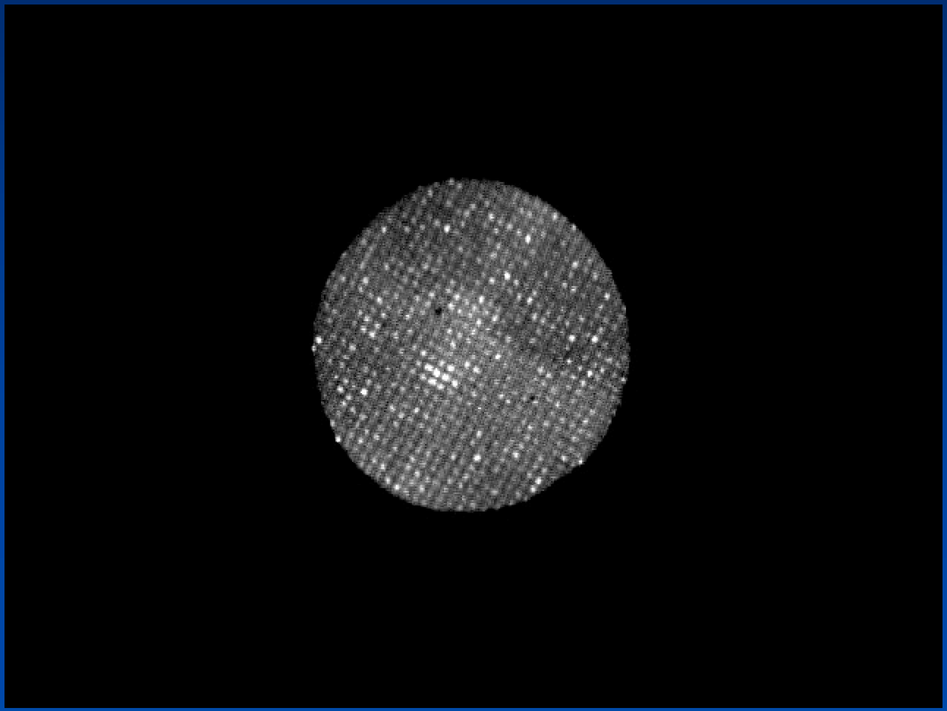
$$P_{1,2} = \frac{(1 - S_{1,2})1000}{d}$$

Note: d is the Talbot plane distance.

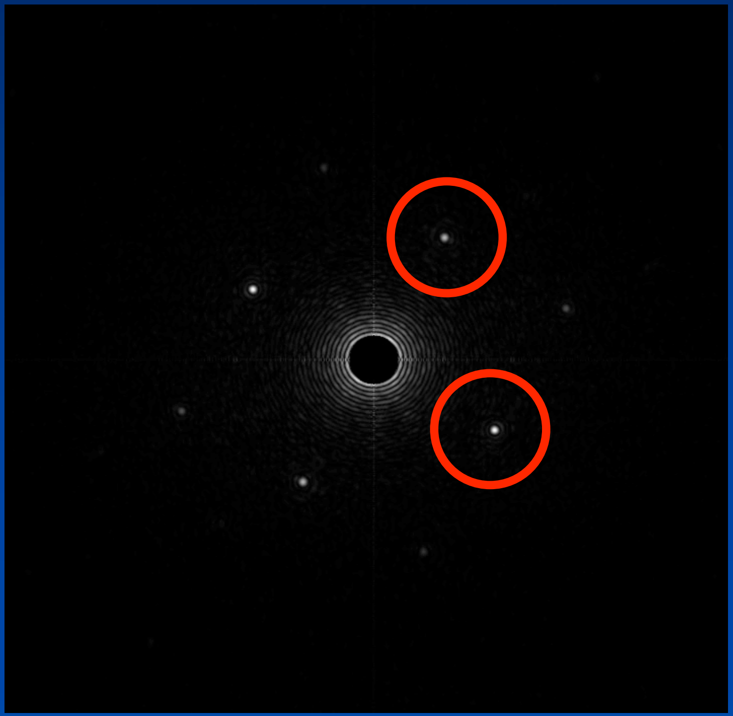
Results for model eye

- 5 exams
- Sphere
  - Mean = -4.26 D
  - SD = 0.004
- Cylinder
  - Mean = -2.90
  - SD = 0.008
- Axis
  - Mean = 92
  - SD = 0

Results for 46 year old male



Spatial domain interferogram



MTF with peaks

Results for 10 exams

Trial	Sphere	Cylinder	Axis
0	0.02	-1.60	69.00
1	-0.11	-1.38	69.00
2	0.03	-1.58	67.00
3	-0.10	-1.61	71.00
4	-0.07	-1.47	71.00
5	-0.02	-1.38	70.00
6	-0.03	-1.55	69.00
7	-0.09	-1.44	68.00
8	-0.28	-1.26	68.00
9	-0.28	-1.23	69.00
mean	**-0.09**	**-1.45**	**69.10**
sd	**0.11**	**0.14**	**1.29**

Discussion

- The sphere, cylinder and axis can be extracted from the MTF of the interferogram of a Talbot moiré wavefront sensor
- Processing time is very fast: 46 ms on a 3.6 GHz PC
  - Fast enough for real time acquisition and display

Conclusion

- MTF calculation method is fast and simple means to process Talbot moiré wavefront sensor images
- May be especially helpful for real-time display of refraction
- May also be helpful in augmenting or providing a quality control check for a full wavefront reconstruction

Thank you!