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Ocular Wavefront Error Representation

A common language for a special community

Charles Campbell
Expressing the refractive error of the eye in the past

• 1280-1860  Really no plan at all
  – The eyes were thought of as ‘weak’ or ‘old’
  – The correcting lenses were thought of as having varying amounts of ‘strength’

• A single value was all that was ever really needed
Expressing the refractive error of the eye in the past

- ~1860 - Astigmatism was slowly accepted as a common visual problem that could be corrected and the need arose to specify more than one value for the refractive error of the eye and corrective means.

- The diopter (D) is introduced (Monoyer 1872) as a more convenient way of expressing the refractive value of a lens.
  
  \[ \text{Diopter} = \frac{1}{\text{focal length in meters}} \]

SPHERE, CYLINDER and AXIS become the common language for describing refractive error - 3 values were sufficient to describe correctable refractive error.
Higher order aberrations

• Known to the physics and optics communities since the time of Descartes, Huygens and Newton
• The full concept of the wavefront developed during 19th century following pioneering work of Young (1801) and Fresnel (1818)
• Description of aberrations codified by Seidel (1857)
• F. Zernike introduces a complete set of orthonormal polynomials to describe aberrations of any complexity (1934)
Higher order aberrations of the eye

- Higher order aberrations of the eye measured in a research setting during most of the 20th century

- As the ability of measure and correct higher order aberrations of the eye becomes a clinical possibility in the last decade of the 20th century a need arises for a good way to describe them
New Capabilities for Measuring Ocular Aberrations Call for a New Way to Describe Aberrations to One Another

- Ways to classify and quantify aberrations
- Common notation
- Common ways to communicate results
Clear Communications Through Standardization

- 1999 - OSA task force formed
- 2000 - OSA task force issues “Standards for Reporting the Optical Aberrations of Eyes”
- 2002 - ANSI Z80 forms committee to create a U.S. national standard based on the OSA standard
- 2002 - ANSI Z80.28 - Methods for reporting optical aberrations of eyes - authorized and work begins
- 2004 - ANSI Z80.28 becomes an American National Standard
- 2005 - ISO TC172/SC7 - Ophthalmic optics and instruments - work begins to create a world standard for Reporting Aberrations of the Human Eye
Use of Zernike polynomial functions to express wavefront error

• Drawing from the success of Zernike polynomials in astronomy and physical optics to express complex wavefronts, the Zernike method was chosen to express the wavefront error of the eye

• It was found, however, that there was not a standardized way to express Zernike functions and that they did not necessarily conform to ophthalmic practices
Issues to Address to Standardize Zernike polynomial representation of aberrations of the eye

- Notation
- Normalization
- Coordinate system
- Aperture size
- Centration
- Display and transmission of coefficients
Several authors, authorities - each with a different notation – something of a mess

- Zernike notation
- Born & Wolf
- U of Arizona
- Noll
- Malacarra
- Zemax
- OSA
- ISO
- Z₆
- Z₃⁻³
- U₃
- Z₁₁
- Z₉
Two index notation method

- Zernike functions are specified by 2 indices
  - $n$ - the radial index
  - $m$ – the meridional index

**Notation of choice** - $Z_n^m$

**Alternate notation** - $Z(n,m)$
Single index notation

• Best thought of as an ordering method

\[ Z_j \quad \text{where:} \quad j = \frac{n(n + 2) + m}{2} \]
Single or Double Index?

$\mathbb{Z}_9$ ?

$\mathbb{Z}_{12}$ ?

$\mathbb{Z}_{15}$ ?
Single or Double Index?

\[ Z_9 = Z_3^3 \]

\[ Z_{12} = Z_4^0 \]

\[ Z_{15} = Z_5^{-5} \]
Zernike Coefficient Normalization

- Zernike functions have 3 terms
  - Normalization $N_n$
  - Radial $R(\rho)_n^{\mid m\mid}$
  - Meridional $M(m\theta)$

$$Z_n^m(\rho, \theta) = N_n R(\rho)_n^{\mid m\mid} M(m\theta)$$

The value $N$ must be chosen
Zernike Coefficient Normalization

- If N is chosen to equal 1, the coefficients are said to be “un-normalized”
- If N is chosen to equal \( \sqrt{(2 - \delta_m)(n + 1)} \), the coefficients are said to be normalized and

The square root of the sum of the squares of the coefficients is equal to the RMS wavefront error

Choice - Normalized
Zernike Coordinate System

Zernike polynomials raised in astronomy find themselves in ophthalmic optics where things are a little different – to say the least.
Coordinate systems

- Astronomers
- Geographers
- Sailors

- Mathematicians
- Physicists
- Ophthalmic professions
  - ISO 8429:1986

Choice - *Ophthalmic*
**Aperture**

- Changing aperture size changes coefficient values for the same wavefront

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**Effect on coefficients of changing pupil diameter**

<table>
<thead>
<tr>
<th>Zernike terms</th>
<th>Coefficient value (microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, -1)</td>
<td>-3.500</td>
</tr>
<tr>
<td>(1, 1)</td>
<td>-3.000</td>
</tr>
<tr>
<td>(2, -1)</td>
<td>-2.500</td>
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<td>(2, 0)</td>
<td>-2.000</td>
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<tr>
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<tr>
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<tr>
<td>(6, 4)</td>
<td>8.500</td>
</tr>
<tr>
<td>(6, 6)</td>
<td>9.000</td>
</tr>
</tbody>
</table>

**Green bars = 4 mm pupil  Red bars = 6 mm pupil**

**The aperture diameter must ALWAYS be included with the Zernike coefficient values reported**
Centration

- A change in coordinate center changes Zernike coefficient values

- Choice – Pupil center

Green - centered

Red - decentered 0.5 mm

Effect on Zernike coefficients of de-centration

Zernike terms

Coefficient values (microns)
Centration

- A change in coordinate center changes Zernike coefficient values

Green - centered

Red - decentered 0.5 mm

Choice – Pupil center
Centration

- A change in coordinate center changes Zernike coefficient values

Choice – Pupil center

Green - centered

Red - decentered 0.5 mm
Effects of de-centered corrections

- The original errors are completely corrected
- New aberrations are introduced
- Each original Zernike term introduces terms of lower order
- The magnitude of the introduced aberrations are proportional to the magnitude of the original term and to the magnitude of de-centeration
- Example – de-centered sphere and cylinder (2\textsuperscript{nd} order) introduce prism (tilt – 1\textsuperscript{st} order)
Zernike Coefficient Tables

- **APERTURE DIAMETER** used to create the coefficients, expressed in millimeters, is always included.

- **NORMALIZED COEFFICIENTS**, expressed in microns of wavefront error, are used.

- List coefficients in **STANDARDIZED SINGLE INDEX ORDER**.

- **STANDARDIZED DOUBLE INDEX ZERNIKE SYMBOLS** used to label the coefficients.

<table>
<thead>
<tr>
<th>Pupil</th>
<th>6.00</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>$Z_7$</td>
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<td>$Z_8$</td>
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<tr>
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<tr>
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<tr>
<td>$Z_{12}$</td>
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<tr>
<td>$Z_{13}$</td>
<td>0.146</td>
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<tr>
<td>$Z_{14}$</td>
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<tr>
<td>$Z_{15}$</td>
<td>-0.030</td>
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<tr>
<td>$Z_{16}$</td>
<td>0.067</td>
</tr>
<tr>
<td>$Z_{17}$</td>
<td>0.005</td>
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<tr>
<td>$Z_{18}$</td>
<td>0.025</td>
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<tr>
<td>$Z_{19}$</td>
<td>-0.022</td>
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<tr>
<td>$Z_{20}$</td>
<td>0.023</td>
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<tr>
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<tr>
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<td>$Z_{25}$</td>
<td>-0.037</td>
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<tr>
<td>$Z_{26}$</td>
<td>-0.011</td>
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</table>
Reporting Zernike terms by magnitude and orientation

• Zernike terms with non-zero meridional indices are vectors and both components should be given when reporting results.

• Just as we would not report astigmatism by giving only the 90/180° component, other vector-like aberrations such as coma and trefoil should have both components reported or given in the form of a magnitude and orientation (axis).
To determine the magnitude and orientation of any given aberration that has an angular frequency other than zero, the sine and cosine modes for the aberration of interest are added. For example, vertical coma (sine mode) and horizontal coma (cosine mode) are added together to establish the total magnitude and orientation of coma.
Zernike notation-
magnitude/axis form

- Double index notation
- Indices written as subscripts
- Meridional index is unsigned

Term: $Z_{n,m}$  Coefficient: $c_{n,m}$

Astigmatism: $Z_{2,2}$
Coma: $Z_{3,1}$
Trefoil: $Z_{3,3}$
Zernike Coefficient Tables in Magnitude/Axis Form

- **APERTURE DIAMETER** used to create the coefficients, expressed in millimeters, is always included.

- **NORMALIZED COEFFICIENTS**, expressed in microns of wavefront error, are used.

- List coefficients in **GROUPS BY RADIAL INDEX ORDER, WITHIN GROUPS BY MERIDIONAL INDEX ORDER**.

- **STANDARDIZED DOUBLE INDEX ZERNIKE SYMBOLS** used to label the coefficients.

<table>
<thead>
<tr>
<th></th>
<th>magnitude</th>
<th>axis</th>
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</thead>
<tbody>
<tr>
<td>pupil</td>
<td>6.000</td>
<td></td>
</tr>
<tr>
<td>$Z_{00}$</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>$Z_{1,1}$</td>
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<tr>
<td>$Z_{2,0}$</td>
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<tr>
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<tr>
<td>$Z_{5,5}$</td>
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</tr>
<tr>
<td>$Z_{6,0}$</td>
<td>-0.008</td>
<td></td>
</tr>
<tr>
<td>$Z_{6,2}$</td>
<td>0.033</td>
<td>3</td>
</tr>
<tr>
<td>$Z_{6,4}$</td>
<td>0.040</td>
<td>39</td>
</tr>
</tbody>
</table>
Zernike coefficients in bar chart form

Zernike coefficients presented as a bar chart

Zernike coefficient magnitudes present as a bar chart
Presentation of wavefront information

• How can we present wavefront information in an intelligible, uniform manner?
Display of aberrations

- Zernike coefficients
- Wavefront error contours maps
Displays from various wavefront refractors
ANSI Unified Scale

- Wavefront value equals zero at the pupil center
- 21 color incremental scale - red most positive error, green zero error - blue most negative error
- Standardized scale increments - 0.1, 0.2, 0.5, 1.0 microns
- Pupil center marked with diameter and meridians indicated
ANSI Unified Scale
Communication of results

• Zernike coefficients
  – Single index ordered list starting with aperture diameter in mm, normalized coefficients in microns
  – Magnitude/axis table with aperture diameter in mm, normalized coefficients in microns

• Deflection information
  – 2 gradient position arrays in mm with a coordinate system with the pupil center as its origin
  – 2 gradient component arrays

• Wavefront error magnitude information
  – 2 magnitude position arrays in mm with a coordinate system with the pupil center as its origin
  – 1 array of signed magnitude values in microns
Information on using Zernike polynomials

- Calculation of coefficients using aberrometer data
- Accounting for rotation and translation of the coordinate system
- Accounting for changes in pupil size
- Conversion from non-standard Zernike notation
- Mathematical forms of the Zernike polynomials with their common names
What you will find in ANSI Z80.28 - Standardized items

- Standardized definitions and terms with particular attention to Zernike polynomial functions
- Standardized coordinate system definition
- Standardized methods of communicating wavefront information in the form Zernike coefficient sets, wavefront error gradients and wavefront errors
- Standardization of wavefront error displays
What you will find in ANSI Z80.28 - Useful information

• Information on creating Zernike coefficient sets from wavefront gradient data
• Information on compensating wavefront information as pupil size, centration and orientation change
• A list of Zernike polynomial functions with their mathematical forms and common names
• Information on converting non-standard Zernike coefficient sets to the standard form
Topics for future work

• Creation of an International Standard for Reporting aberrations of the human eye
• Inclusion of new visual metrics that fully use the new information available in a clear, clinically relevant fashion
• Presentations of point spread functions
• Standardization of the aberrometers themselves
Comparison of measurements

- equivalency of aberrometers
- test devices

Still an open issue