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Extracting wavefront error from S/H images using Spatial Demodulation

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Outline

- Traditional Shack-Hartmann (SH) processing
- Fourier Transform (FT) processing of SH images
- Spatial Demodulation (SD) processing of SH images
- Simulation and exam examples
- Discussion
SH wavefront sensor

Micro lens array  Image plane

Wavefront propagation
SH wavefront sensor

Incident Plane wavefront

Micro lens array

Image plane

Reference spots in regular grid on image plane

Wavefront propagation
SH wavefront sensor

Micro lens array  Image plane

Wavefront propagation
SH wavefront sensor

Micro lens array

Image plane

Aberrated wavefront

Aberrated spots in irregular grid on image plane

Wavefront propagation
Traditional SH Processing

- Find centroid corresponding to each micro lens
- Deviation $dx$, $dy$ of detected centroid from reference spots yields wavefront gradient
- Reconstruct wavefront from gradient (e.g., Zernike or Fourier)

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WF gradient found from spot deviation...

Traditional SH Processing

Micro lens array Image plane

$dW(x,y) / dy = -\frac{\text{del}Y}{f}$

$dW(x,y) / dx = -\frac{\text{del}X}{f}$
Fourier SH Processing

- Compute Fourier Transform of SH image
- Isolate region of interest (band-pass filter) and shift to center
- Compute inverse Fourier Transform and compute complex angle to yield wrapped phase
- Unwrap phase to yield wavefront gradient
- Reconstruct wavefront from gradients (e.g., Zernike or Fourier)

Looking at same neighborhood as before...

Micro lens array Image plane

Fourier SH Processing

$f$

$Y$

$Y'$

$delY$

$f$
Warp function is key...

Wavefront warps lens center locations from MLA plane to the image plane.

\[ g_0(Y) \]
\[ g_1(Y') = g_0(Y' - \text{del}Y) \]

\[ \text{del}Y = -\frac{dW(y)}{dy} \times f \]

\[ g_1(y') = g_0\left( y' + \frac{dW(y)}{dy} \times f \right) \]
If we warp the coordinates of a reference spot pattern, what happens to its Fourier Transform?

First, we look at just the reference spots...
Spots and FT of Spots

\[ g_0(x, y) = \left[ \frac{1}{2} + \frac{1}{2} \cos\left( \frac{2\pi}{p_x} x \right) \right] \times \left[ \frac{1}{2} + \frac{1}{2} \cos\left( \frac{2\pi}{p_y} y \right) \right] \]

\[ G_0(u, v) = \frac{1}{4} \delta(0, 0) + \frac{1}{8} \left[ \delta \left( u - \frac{1}{p_x}, v \right) + \delta \left( u + \frac{1}{p_x}, v \right) \right] \ldots \]
Reference spots: $g_0(x,y)$
FT of reference spots: $G_0(u,v)$
Now, warped spots...

\[ g_1(x, y) = g_0 \left[ x + A(x, y), y + B(x, y) \right] \]

\[ = \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{2\pi}{p_x} (x + A(x, y)) \right) \right] \times \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{2\pi}{p_y} (y + B(x, y)) \right) \right] \]

\[ A(x, y) = \frac{dW_2(x, y)}{dx} \times f, \quad B(x, y) = \frac{dW_2(x, y)}{dy} \times f \]
Our goal is to recover $A(x,y)$ from the low-pass signal $\exp(j2\pi A(x,y))$.

Similarly, for $B(x,y)$.
Sample image of warped spots...
...and its Fourier transform
Fourier SH Processing

Regions of interest in FT of spots...

This region has info about $dW/dy$

This region has info about $dW/dx$
Constraint on WF...

- **WF Requirement:**
  - Wavefront spectrum must be band limited.
  - In particular, the wavefront slope frequency is low compared to MLA spot frequency.

- **Spot pattern requirement:**
  - The width of the spot pattern should be large relative to the spot pitch – we should have quite a few spots across the image.
Phase unwrapping

- Why is it required?
  - Because the wavefront slope appears in the recovered exponential and “wraps” around every $2\pi$.

- How is it done?
  - If no “residues”, unwrapping is simple.
  - If residues are present, unwrapping can be difficult (see ¹)

Simple unwrapping

Unwrapped function

Wrapped function
Simple unwrapping...

- Set $U(0) = W(0)$
- For $n=1$ to the end do the following:
  \[ D = W(n) - W(n - 1) \]
  - If $D < -\pi$ then $D = D + 2\pi$
  - If $D > \pi$ then $D = D - 2\pi$
  - $U(n) = U(n-1) + D$

$U = \text{unwrapped}$
$W = \text{wrapped}$
Test for Residue

For a continuous surface (wavefront slopes) the sum of the deltas around a closed curve should be zero.

If the sum is non-zero, a residue exists and the simple unwrapping method is not valid. Examples given below.
FT processing steps for Plane wave...
Contours in input image
Fourier SH Processing

FT of input image (DC suppressed)
Fourier SH Processing
Shift spectrum to center ROI’s...

Shift to center ROI for $dW/dX$
Shift to center ROI for $dW/dY$
Shift spectrum to center ROI’s...

- Shift to center ROI for $\frac{dW}{dX}$
- Shift to center ROI for $\frac{dW}{dY}$
Fourier SH Processing

ROI’s isolated...

ROI for $dW/dX$  
ROI for $dW/dY$
Wrapped phase

Wrapped phase for $dW/dX$  
Wrapped phase for $dW/dY$
Fourier SH Processing

Unwrapped phase

Unwrapped phase for \( \frac{dW}{dX} \)

Unwrapped phase for \( \frac{dW}{dY} \)
Reconstructed wavefront

Note zero aberrations since plane wave.
Spatial demodulation

- Similar to FT method, but does not require FFTs
- Multiply spots image by complex exponential to shift the desired neighborhood to origin in the frequency domain
- Low-pass filter to isolate the desired frequency band (box filter is fast)
- Unwrap the result
- Reconstruct the wavefront

1Talmi and Ribak, Direct demodulation of Hartmann-Shack patterns, JOSA; vol 21, No 4, 632-9, 2004.
The FT modulation relation shows that multiplication by a complex exponential in the spatial domain shifts the Fourier Transform in the spectral domain.
Low-pass box filter

- For spatial demodulation, low-pass filter is performed via convolution
- To provide efficient calculation, a box filter is used -- (all coefficients equal)
- Using a sliding sum, the filter is computed using only 4 adds per output sample (no multiplies)
- We use two passes to provide a “cleaner” frequency response
Spatial Demodulation SH Processing

Triangle $\rightarrow$ Sinc$^2$

$$\text{rect}(ax) \ast \text{rect}(ax) = \text{triangle}(ax) \leftrightarrow_{\text{FT}} \left( \frac{1}{|a|} \text{sinc} \left( \frac{s}{a} \right) \right)^2$$

Triangle$(x, 0.5)$

Sinc$(x, 0.5)^2$
Comparison of FT domain and Spatial domain

- Shift spectral region to center
  - FT: Compute FT and shift complex array
  - SD: Multiply spots image by complex exponential

- Remove unwanted frequencies
  - FT: Zero areas outside ROI
  - SD: Apply low-pass filter via convolution

- Obtain wrapped wavefront derivatives
  - FT: Take IFT and calculate complex phase
  - SD: Calculate complex phase

- Rest of processing the same...
Examples

- Simulated spot images
  - Astigmatic wavefront
  - High dynamic range (± 20 D)
  - High resolution (-0.01 D)
  - Third-order aberrations (trefoil & coma)
- Eye image
  - Large amount of background noise
- Simulated “bad” exam
  - Not sufficiently band limited
Astigmatic wavefront

-5 S -2 C x 17
Astigmatic wavefront

Detected contours
Astigmatic wavefront

MTF of input image
Regions of interest in MTF
Astigmatic wavefront

Wrapped phase

dW/dX
dW/dY
Unwrapped phase

Astigmatic wavefront

dW/dX

dW/dY
Astigmatic wavefront

Reconstructed wavefront

Higher-order aberrations

Whole eye wavefront

Sph  -5.00 D
Cyl  -2.00 D
Axis 17°
LO RMS 7.13 μm
HO RMS 0.01 μm
TOT RMS 7.13 μm
Pupil 5.66 mm

ANSI Z80.28
High dynamic range

Examples

-20 D          + 20 D
Detected contours

-20 D  + 20 D

High dynamic range
High dynamic range

MTF of input image

-20 D

+ 20 D
Regions of interest in MTF

-20 D

+ 20 D

High dynamic range
High dynamic range

Wrapped x-gradient

-20 D

+ 20 D
High dynamic range

Unwrapped x-gradient

-20 D + 20 D
High dynamic range

Wrapped y-gradient

-20 D  + 20 D
Unwrapped y-gradient

High dynamic range

-20 D  + 20 D
High dynamic range

Reconstructed wavefront

-20 D  + 20 D

Higher-order aberrations

Sph  -20.00 D
Cyl -0.00 D
Axe  107°
LO RMS 16.63 μm
HO RMS 0.05 μm
TCT RMS 16.93 μm
Pupil 4.94 mm

ANSI Z80.28
High dynamic range

Zernike bar graph

-20 D  +  20 D
High Resolution: -0.01D
Detected contours
MTF of input image
Regions of interest in MTF
High resolution

Wrapped phase

dW/dX
dW/dY
High resolution

Unwrapped phase

dW/dX
dW/dY
Reconstructed wavefront

Higher-order aberrations

Wavefront Zernikes

ANSI Z80.28
Third-order aberrations
Trefoil and Coma...
Third-order aberrations

Detected contours

Trefoil

Coma
Third-order aberrations

MTF of input image

Trefoil

Coma
Regions of interest in MTF

Third-order aberrations

Trefoil

Coma
Third-order aberrations

Wrapped phase for $dx$

Trefoil

Coma
Third-order aberrations

Unwrapped phase for $dx$

Trefoil

Coma
Third-order aberrations

Wrapped phase for dy

Trefoil

Coma
Third-order aberrations

Unwrapped phase for dy

Trefoil

Coma
Third-order aberrations

Reconstructed wavefront

Trefoil

Coma

Higher-order aberrations

Whole eye wavefront

Sph  -0.00 D
Cyl  -0.00 D
Axe  10°
LO RMS 0.00 μm
HO RMS 1.80 μm
TCT RMS 1.80 μm
Pupil 5.00 mm

ANSI Z80.28
Third-order aberrations

Zernike bar graph

Trefoil

Coma

Wavefront Zernikes

Whole eye wavefront

Value um

Coefficient

Wavefront Zernikes

Whole eye wavefront

Value um

Coefficient
Examples

Real eye
MTF of input image
Regions of interest in MTF
Wrapped phase

dW/dX

dW/dY

Real eye
Real eye

Unwrapped phase

\( \frac{dW}{dX} \)  \( \frac{dW}{dY} \)
Real eye

Reconstructed wavefront

SD Reconstruction

-3.37 – 2.07 x 125

Spot Centroid Reconstruction

-3.12 – 1.80 x 125
What do bad cases look like?

- $C_{10} = C_{11} = 2$ microns
  - Not properly band limited
C10 = C11 = 2 microns

Not band limited
Detected contours

Not band limited
Not band limited

MTF of input image
Wrapped phase

Not band limited

dW/dX
dW/dY
Unwrapped phase

FAIL TO UNWRAP

Not band limited

dW/dX   dW/dY
Discussion

- Compared to SH centroid method the FT/SD method:
  - Does not require finding spot centroids
  - Easily allows spots to move outside their initial aperture region
  - Requires phase unwrapping processing

- The FT technique is especially suited to large arrays and large aberrations
Discussion...

- Compared to FT method the SD method:
  - Does not require FT of arrays
  - Allows handling unwrapping problems locally
Summary

- FT/SD method provides another tool for finding wavefront from a SH image
- For simulated images the FT/SD method:
  - Has a large dynamic range
  - Has high resolution
- Caution: Errors occur when the bandwidth of the wavefront exceed the threshold imposed by the MLA lens spacing
Thank you!
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