Methods for Measuring Ocular Wavefront Error

Edwin J. Sarver, PhD
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San Francisco, CA
USA
Topics

- What is wavefront error?
- How is raw aberration data (wavefront slopes) obtained?
  - Measurement using In going rays
  - Measurement using Out going rays
Here are some rays from a distant object propagating into a perfect eye. The rays come to a perfect focus at the retina.
What is wavefront error?

A wavefront is perpendicular to the rays. Here is an incident plane wave.
What is wavefront error?

Plane and spherical wavefronts

Propagation

Perfect eye

And a converging spherical wave.
What is wavefront error?

For a perfect eye, the wavefront and the reference sphere are equal.
What is wavefront error?

**In going wavefront error**

For an aberrated eye, the actual wavefront and the reference are not equal. Here we see the wavefront aberrations for rays directed into the eye.
What is wavefront error?

**Outgoing wavefront error**

We can also define the wavefront aberrations for rays directed out of the eye. In this case the reference is a plane.
Wavefront error

- By OSA convention, wavefront error is measured in the plane of the eye’s pupil
- Wavefront error can be measured using either in going or out going rays
Measurement using in going rays

- Image space aberrations
- Main techniques
  - Adjust incident marginal ray to be coincident with reference ray
  - Measure retinal distance from reference ray to retinal ray height (transverse ray aberration)
  - Principle of retinoscopy
Adjusting incident ray angle
Incident chief ray and measurement ray. For an aberrated eye, the measurement ray does not intersect the chief ray at the retina.
Incident ray angle is adjusted to bring two points together at retina. This adjustment could be subjective or objective.
Aberrated eye

The ray adjustment angle gives us the wavefront slopes. Wavefront slopes can be measured at several points in the entrance pupil.
How are wavefront slope data turned into wavefront measurements?
Analysis to generate wavefront

- Given the wavefront slope values we can generate a representation of the wavefront.
- Zernike expansion fit
- Fourier fit
- Table integration (Spline)
General steps to fit wavefront

- Determine a set of wavefront slopes over the pupil
- For a given wavefront surface:
  - Use each point to constrain the surface parameters (e.g., Zernike coefficients)
  - Solve a linear system of equations for the surface parameters
Observing transverse ray aberration
For the aberrated eye, an off axis ray will not intersect the chief ray at the retina. The retinal ray height is called the transverse ray aberration.
We can calculate the wavefront slope according to its relationship to the transverse ray aberration. Note the dependence upon the length $L$.

$$\frac{\partial W(x, y)}{\partial x} = n \frac{x_C - x_R}{L}$$

Several such rays can be evaluated to generate the wavefront slopes over the entire entrance pupil.
Observing one or more retinal rays

- Can scan the entrance pupil to collect wavefront slopes sequentially
- Can project multiple incident rays and capture a retinal image to calculate wavefront slopes all at once
Simultaneous rays...

For simultaneous measurement, we project an array of rays into the eye.
Simultaneous rays...

Find transverse ray aberration for each ray, then use ray/wavefront relation to determine wavefront slopes.
Principle of retinoscopy
In retinoscopy, the retina is scanned with a beam of light.
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For all three cases of hyperopic, emmetropic, and myopic eyes, the retinal scan is in the same direction.
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Scan for hyperopic, emmetropic, and myopic eyes

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For all three cases of hyperopic, emmetropic, and myopic eyes, the retinal scan is in the same direction.
As the beam is scanned across the retina, it is detected by sensors.
The aperture of the beam detection system is conjugate to the emmetropic retina.
For a fixed location on the sensor array, the input beam must change angles from negative to positive for eyes ranging from hyperopic to myopic.
• For a fixed point on the sensor plane, we have a fixed point in the entrance pupil.
• The input scan angle is measured corresponding to this sensor point.
• The correction power at this pupil location can then be determined.
The entrance pupil is scanned on a meridian by meridian basis to obtain the power measurements at several points.
For each location that a power measurement is obtained, we can calculate the corresponding wavefront slope. From this we can compute the wavefront.

\[ \frac{\partial W(r, \theta)}{\partial r} = \frac{D \times r}{1000} \]

\[ d = \frac{1000}{D} \]
Measurement using outgoing rays

- Shack-Hartmann wavefront sensor
- Talbot Moiré Interferometry
Systems which use outgoing rays begin by placing a diffuse source at the retina.
A source beam is directed into the eye to form a diffuse reflection at the retina.
A source beam is directed into the eye to form a diffuse reflection at the retina. This diffuse spot then acts as a source for the outgoing rays.
The wavefront at the eye’s entrance pupil is relayed via the relay lens to the wavefront sensor. The camera then records the output of the wavefront sensor.
Shack-Hartmann wavefront sensor
SH wavefront sensor
SH wavefront sensor

Incident Plane wavefront

Micro lens array  Image plane

Reference spots in regular grid on image plane

Wavefront propagation
SH wavefront sensor

Micro lens array  Image plane

Wavefront propagation
SH wavefront sensor

Micro lens array

Image plane

Aberrated wavefront

Aberrated spots in irregular grid on image plane

Wavefront propagation
WF slope found from spot deviation...

\[
dW(x,y) / dy = -\Delta Y / f
\]

\[
dW(x,y) / dx = -\Delta X / f
\]
Traditional SH Processing

- Find centroid corresponding to each micro lens
- Deviation $dx$, $dy$ of detected centroid from reference spots yields wavefront slope
- Reconstruct wavefront from slopes (e.g., Zernike or Fourier)

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Fourier SH Processing

- Compute Fourier Transform of SH image
- Isolate region of interest (band-pass filter) and shift to center
- Compute inverse Fourier Transform and compute complex angle to yield wrapped phase
- Unwrap phase to yield wavefront slope
- Reconstruct wavefront from slopes (e.g., Zernike or Fourier)


Looking at same neighborhood as before...
Warp function is key...

Wavefront warps lens center locations from MLA plane to the image plane.

\[ g_0(Y) \]
\[ g_1(Y') = g_0(Y' - \text{del}Y) \]

\[ \text{del}Y = -\frac{dW(y)}{dy} \times f \]

\[ g_1(y') = g_0\left(y' + \frac{dW(y)}{dy} \times f \right) \]
If we warp the coordinates of a reference spot pattern, what happens to its Fourier Transform?

First, we look at just the reference spots...
Spots and FT of Spots

\[ g_0(x, y) = \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{2\pi}{p_x} x \right) \right] \times \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{2\pi}{p_y} y \right) \right] \]

\[ G_0(u, v) = \frac{1}{4} \delta(0,0) + \frac{1}{8} \left[ \delta \left( u - \frac{1}{p_x}, v \right) + \delta \left( u + \frac{1}{p_y}, v \right) \right] \ldots \]
Reference spots: \( g_0(x,y) \)
FT of reference spots: $G_0(u,v)$
Now, warped spots...

\[ g_1(x, y) = g_0 \left[ x + A(x, y), y + B(x, y) \right] \]

\[ = \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{2\pi}{p_x} (x + A(x, y)) \right) \right] \times \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{2\pi}{p_y} (y + B(x, y)) \right) \right] \]

\[ A(x, y) = \frac{dW_2(x, y)}{dx} \times f, \quad B(x, y) = \frac{dW_2(x, y)}{dy} \times f \]
Our goal is to recover $A(x,y)$ from the low-pass signal $\exp(j2\pi A(x,y))$. Similarly, for $B(x,y)$. 

\[
G_2(u,v) = \int g(x + A(x,y)) e^{-j2\pi(ux + vy)} \, dx \, dy
\]

\[
= \int g(x) e^{j2\pi A(x,y)} e^{-j2\pi(ux + vy)} \, dx \, dy
\]

Our goal is to recover $A(x,y)$ from the low-pass signal $\exp(j2\pi A(x,y))$. Similarly, for $B(x,y)$. 

Fourier SH Processing
Sample image of warped spots...
...and its Fourier transform
Regions of interest in FT of spots...

This region has info about $dW/dy$

This region has info about $dW/dx$
Constraint on WF...

- **WF Requirement:**
  - Wavefront spectrum must be band limited
  - In particular, the wavefront slope frequency is low compared to MLA spot frequency

- **Spot pattern requirement:**
  - The width of the spot pattern should be large relative to the spot pitch – we should have quite a few spots across the image
Phase unwrapping

**Why is it required?**
- Because the wavefront slope appears in the recovered exponential and “wraps” around every $2\pi$.

**How is it done?**
- If no “residues”, unwrapping is simple.
- If residues are present, unwrapping can be difficult (see ¹)

Simple unwrapping

Unwrapped function

Wrapped function
For a continuous surface (wavefront slopes) the sum of the deltas around a closed curve should be zero.

If the sum is non-zero, a residue exists and the simple unwrapping method is not valid.
Fourier SH Processing

FT processing steps for Plane wave...
Contours in input image
FT of input image (DC suppressed)
Shift spectrum to center ROI’s...
Fourier SH Processing

Shift spectrum to center ROI’s...

- Shift to center ROI for dW/dX
- Shift to center ROI for dW/dY
Fourier SH Processing

ROI’s isolated...

ROI for $dW/dX$

ROI for $dW/dY$
Fourier SH Processing

Wrapped phase

Wrapped phase for \(\frac{dW}{dX}\)

Wrapped phase for \(\frac{dW}{dY}\)
Fourier SH Processing

Unwrapped phase

Unwrapped phase for \( \frac{dW}{dX} \)

Unwrapped phase for \( \frac{dW}{dY} \)
Reconstructed wavefront

Note zero aberrations since plane wave.
More interesting examples...

- Simulated spot images
  - Astigmatic wavefront
  - High dynamic range (± 20 D)
  - Third-order aberrations (trefoil & coma)
Examples

Astigmatic wavefront
-5 S -2 C x 17
Detected contours
Astigmatic wavefront

FT of input image
Astigmatic wavefront

Regions of interest in FT
Astigmatic wavefront

Wrapped phase

dW/dX
dW/dY
Astigmatic wavefront

Unwrapped phase

\[ \frac{dW}{dX} \quad \frac{dW}{dY} \]
Astigmatic wavefront

Reconstructed wavefront

Higher-order aberrations

Whole eye wavefront

Sph  -5.00 D
Cyl  -2.00 D
Axis 17°
LO RMS 7.13 μm
HO RMS 0.01 μm
TOT RMS 7.19 μm
Pupil 5.66 mm

ANSI Z80.28
Examples

High dynamic range

-20 D  + 20 D
High dynamic range

 Detected contours

-20 D

+ 20 D
High dynamic range

FT of input image

-20 D  + 20 D
High dynamic range

Regions of interest in FT

-20 D

+ 20 D
High dynamic range

Wrapped x-gradient

-20 D

+ 20 D
High dynamic range

Unwrapped x-gradient

-20 D + 20 D
High dynamic range

Wrapped y-gradient

-20 D

+ 20 D
High dynamic range

Unwrapped y-gradient

-20 D + 20 D
High dynamic range

Reconstructed wavefront

-20 D

+ 20 D

Higher-order aberrations

Sph: -20.00 D
Cyl: -0.00 D
Axis: 107°
LO RMS: 16.93 μm
HO RMS: 0.05 μm
TCT RMS: 19.93 μm
Pupil: 4.94 mm

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Higher-order aberrations

Sph: 20.00 D
Cyl: -0.00 D
Axis: 123°
LO RMS: 22.42 μm
HO RMS: 0.00 μm
TCT RMS: 22.42 μm
Pupil: 5.57 mm

ANSI Z80.28
Third-order aberrations
Trefoil and Coma...

Trefoil

Coma
Third-order aberrations

Detected contours

Trefoil

Coma
Third-order aberrations

FT of input image

Trefoil

Coma
Third-order aberrations

Regions of interest in FT

Trefoil

Coma
Third-order aberrations

Wrapped phase for $dx$

Trefoil

Coma
Third-order aberrations

Unwrapped phase for dx

Trefoil

Coma
Third-order aberrations

Wrapped phase for dy

Trefoil

Coma
Unwrapped phase for dy

Trefoil

Coma
Third-order aberrations

Reconstructed wavefront

Trefoil

Coma

Higher-order aberrations

Sph: -0.00 D
Cyl: -0.00 D
Axe: 10°
LO RMS: 0.03 μm
HO RMS: 1.80 μm
TOT RMS: 1.80 μm
Pupil: 5.90 mm

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Higher-order aberrations

Sph: -0.01 D
Cyl: -0.00 D
Axe: 1°
LO RMS: 0.01 μm
HO RMS: 2.23 μm
TOT RMS: 2.23 μm
Pupil: 6.23 mm

ANSI Z80.28
A close relative to Fourier processing of SH image is spatial demodulation processing.
Spatial demodulation

- Similar to FT method, but does not require FFTs
- Multiply spots image by complex exponential to shift the desired neighborhood to origin in the frequency domain
- Low-pass filter to isolate the desired frequency band
- Unwrap the result
- Reconstruct the wavefront

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1Talmi and Ribak, Direct demodulation of Hartmann-Shack patterns, JOSA; vol 21, No 4, 632-9, 2004.
The FT modulation relation shows that multiplication by a complex exponential in the spatial domain shifts the Fourier Transform in the spectral domain.
Low-pass filter

- For spatial demodulation, low-pass filter is performed via convolution
- Filter selection can affect final performance of system
- Want frequency response of filter band limited to prevent aliasing
- Want impulse response of filter to have limited length to reduce width of corrupt region inside pupil
Comparison of FT domain and Spatial domain

- Shift spectral region to center
  - FT: Compute FT and shift complex array
  - SD: Multiply spots image by complex exponential

- Remove unwanted frequencies
  - FT: Zero areas outside ROI
  - SD: Apply low-pass filter via convolution

- Obtain wrapped wavefront slopes
  - FT: Take IFT and calculate complex phase
  - SD: Calculate complex phase

- Rest of processing the same...
Talbot Moiré Interferometry
Ronchi Ruling
Ronchi Grid
Ronchi Grid Parameters

\[ p \]

\[ p/2 \]
Ronchi Grid Alternative

- Instead of two-dimensional array of square apertures, one could use a two-dimensional array of circle apertures (Hartmann screen).
Ronchi Grid as Transmission Function

Wavefront propagation

G1  d  Observation plane
For simplicity, let’s model amplitude transmission$^1$ as a sinusoid...

\[ T(x, y) = \frac{1}{2} \left[ 1 + \cos \left( \frac{2\pi}{\lambda} x \right) \right] \]

Fresnel diffraction leads to intensity distribution given by...

\[ I(x, y) = \frac{1}{4} \left[ 1 + 2A \cos \left( \frac{2\pi}{p} x \right) + \cos^2 \left( \frac{2\pi}{p} x \right) \right] \]

where

\[ A = \cos \left( \frac{\pi \lambda}{p^2} d \right) \]

A = ± 1 provides maximum contrast and exact replica of transmission plane. Corresponding $d$ location is called **Talbot plane**.

$$A = \cos \left( \frac{\pi \lambda}{p^2} d \right)$$
Now we place a second Ronchi Grid at a Talbot distance $d$...
...And rotate the second grid
This superposition of grid patterns leads to a moiré pattern. The combination of the moiré effect with the Talbot distance requirement leads to the name…

Talbot Moiré Interferometer
Fourier domain primary peaks of G1 are...
Fourier domain primary peaks of *rotated* G2 are...
In the spatial domain the propagated transmission of G1 is multiplied by the transmission function at G2.

The equivalent operation in the Fourier domain is convolution…
Two primary peaks of FT of Intensity distribution at G2
What happens when the incident wavefront at G1 has defocus?
With diverging incident wavefront...

\[ P2 = S \times P1 \]

\[ S = \text{period scale factor} = 1 - (D \times d)/1000 \]
Fourier domain for scaled G1...
After modulation by G2...
Apparent rotation due to defocus...
Apparent rotation due to defocus...
Apparent rotation due to defocus...
Instead of pure defocus, what about astigmatic wavefront?
Astigmatic wavefront aligned with system axes...

No defocus

Diverging wavefront in u axis direction

Converging wavefront in v axis direction
Results for 46 year old male

Spatial domain interferogram

FT with peaks
Processing of Talbot Moiré images...

- The system setup for SH and for Talbot Moiré wavefront sensors could be identical (only sensor is different).
- The Fourier or spatial domain processing as described earlier can be used to recover the wavefront slopes for the Talbot Moiré interferometer as well.
Summary

- Two general methods for wavefront measurement
  - In going rays (image space)
  - Out going rays (object space)

- Multiple ways to collect wavefront slope data
  - In going rays
    - Adjusting incident ray angle → wavefront slope
    - Measure transverse ray aberration → wavefront slope
    - Retinoscopy → Measure power → wavefront slope
  - Out going rays
    - SH Spot deviations → wavefront slope
    - SH or TM Fourier processing → wavefront slope
    - SH or TM Spatial demodulation → wavefront slope

- Obtain wavefront by fitting slope to ...
  - Zernike, Fourier, Table, ...
For more information and ideas...

Thank you!