Review of Optics

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Geometrical Optics

Relationships between pupil size, refractive error and blur
Optics of the eye: Depth of Focus

2 mm  4 mm  6 mm
Optics of the eye: Depth of Focus

Focused behind retina

In focus

Focused in front of retina

2 mm  4 mm  6 mm
Demonstration
Role of Pupil Size and Defocus on Retinal Blur

Draw a cross like this one on a page. Hold it so close that is it completely out of focus, then squint. You should see the horizontal line become clear. The line becomes clear because you have used your eyelids to make your effective pupil size smaller, thereby reducing the blur due to defocus on the retina image. Only the horizontal line appears clear because you have only reduced the blur in the horizontal direction.
Computation of Geometrical Blur Size

\[
\text{blur [mrad]} \approx \frac{D}{\text{pupil size [mm]}}
\]

\[
\text{blur [minutes]} \approx \frac{360}{D} \text{ [degrees]}
\]

where D is the defocus in diopters
Application of Blur Equation

• 1 D defocus, 8 mm pupil produces
  27.52 minute blur size ~ 0.5 degrees
Physical Optics

The Wavefront
What is the Wavefront?

parallel beam = plane wavefront

converging beam = spherical wavefront
What is the Wavefront?

parallel beam = plane wavefront

ideal wavefront
defocused wavefront
What is the Wavefront?

**parallel beam** = **plane wavefront**

**ideal wavefront**

**aberrated beam** = **irregular wavefront**
What is the Wavefront?

- diverging beam = spherical wavefront
- aberrated beam = irregular wavefront
- ideal wavefront
The Wave
Aberration
What is the **Wave Aberration**?

diverging beam = spherical wavefront
Wave Aberration Contour Map
Wave Aberration: Defocus
Wave Aberration: Coma
Wave Aberration: Complex Aberration
Zernike Polynomials

- It’s convenient to have a mathematical expression to describe the wave aberration
  - allows us to compute metrics
  - allows to display data in different forms
  - allows us to breakdown data into different components (remove defocus, for example)

- Zernike polynomials are a convenient equation to use to fit wave aberration data for the eye
Zernike Polynomials are not a big deal!

- they are just an equation used to fit data
  - $y=mx+b$ is used to fit linear data
  - $y=ax^2+bx+c$ is used to fit parabolic data
  - $Z(x,y)$ are useful for fitting ocular wave aberration data
what magnitude of each of these shapes is present in my wave aberration data?
Breakdown of Zernike Terms

Coefficient value (microns)

Zernike term

-0.5 0 0.5 1 1.5 2

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1 astig. 2 defocus 3 astig. 4 etc.
5 trefoil 6 coma 7 coma 8 trefoil
9 spherical aberration 10 etc.

2nd order
3rd order
4th order
5th order
The Reason we Measure the Wave Aberration

- **PSF** (point spread function)
- **OTF** (optical transfer function)
- **PTF** (phase)
- **MTF** (contrast)

Image Quality Metrics
The Point Spread Function
The Point Spread Function, or PSF, is the image that an optical system forms of a point source.

The point source is the most fundamental object, and forms the basis for any complex object.

The PSF is analogous to the Impulse Response Function in electronics.
The Point Spread Function

The PSF for a perfect optical system is not a point, but is made up a core surrounded by concentric rings of diminishing intensity.

It is called the Airy disc.
Airy Disk

\[ N = \frac{1.22 \times \Theta}{\alpha} \]

- \( N \Theta \): angle subtended at the nodal point
- \( \Theta \Theta \): wavelength of the light
- \( \alpha \Theta \): pupil diameter
As the pupil size gets larger, the Airy disc gets smaller.

\[
N = \frac{1.22 \times \Theta}{\alpha}
\]

- \(N\Theta\) \quad \text{angle subtended at the nodal point}
- \(\Theta\) \quad \text{wavelength of the light}
- \(\alpha\) \quad \text{pupil diameter}

PSF Airy Disk radius (minutes)

pupil diameter (mm)
Resolution
Rayleigh resolution limit
As the pupil size gets larger, the potential resolution improves

\[ \theta_{\text{min}} = \frac{1.22 \cdot \lambda}{\alpha} \]

- \( \theta_{\text{min}} \) = angle subtended at the nodal point
- \( \lambda \) = wavelength of the light
- \( \alpha \) = pupil diameter
Minutes of arc

20/20

5 arcmin

1 arcmin

20/10

2.5 arcmin
Keck telescope (10 m pupil)

About 4500 times better than the eye!

(0.022 arcseconds resolution)
Convolution
Convolution

$PSF(x,y) \times O(x,y)$
Simulated Images

20/20 letters

20/40 letters
MTF
Modulation Transfer Function
The diagram illustrates the relationship between spatial frequency and contrast for objects with different contrast.

- **Low Contrast Object:**
  - The object has low contrast.
  - The image representation shows a low spatial frequency pattern.

- **Medium Contrast Object:**
  - The object has medium contrast.
  - The image representation shows a medium spatial frequency pattern.

- **High Contrast Object:**
  - The object has high contrast.
  - The image representation shows a high spatial frequency pattern.

The graph at the bottom illustrates the contrast as a function of spatial frequency. The contrast decreases as the spatial frequency increases, indicating a trade-off between these two factors.
**MTF: Cutoff Frequency**

Cut-off frequency

\[ f_{cutoff} = \frac{\alpha}{57.3 \cdot \lambda} \]

Rule of thumb: cutoff frequency increases by \(~30\, \text{c/d for each mm increase in pupil size}\)
Spatial frequency

20/20

5 arcmin

30 cyc/deg

20/10

2.5 arcmin

60 cyc/deg
Phase Transfer Function

- Contains information about asymmetry in the PSF
- Contains information about contrast reversals (spurious resolution)
The Importance of Phase
Relationships Between Wave Aberration, PSF and MTF
The Reason we Measure the Wave Aberration

PSF (point spread function)

OTF (optical transfer function)

PTF (phase)  MTF (contrast)

Image Quality Metrics
The PSF is the Fourier Transform (FT) of the pupil function

$$PSF(x_{\text{FT}}(y)) \neq \left\{-i\frac{\pi}{\lambda}(,)ight\}$$

The MTF is the amplitude component of the FT of the PSF

$$MTF(Amp)(x_{\text{FT}}) \neq FTPSF_{xy} \psi \bullet \psi$$

The PTF is the phase component of the FT of the PSF

$$PTF(Phase)(x_{\text{FT}}) \neq FTPSF_{xy} \psi \bullet \psi$$

The OTF (MTF and PTF) can also be computed as the autocorrelation of the pupil function
Wavefront Aberration

Modulation Transfer Function

Phase Transfer Function

Point Spread Function
Conventional Metrics to Define Imagine Quality
Root Mean Square

\[ RMSW = \sqrt{\frac{1}{A} \sum_{xy} (d_2(y, w))^2} \]

A 2 pupil area

\( W_{x(y, w)} \) wave aberration

\( W_{x(y, a)} \) average wave aberration
Root Mean Square:
Advantage of Using Zernikes to Represent the Wavefront

\[ \text{RMS}_Z = \sqrt{ \sum (Z_{ijk})^2 } \]

- astigmatism term
- defocus term
- astigmatism term
- trefoil term

\ldots
Strehl Ratio

\[
Strehl \text{ Ratio} = \frac{H_{\text{eye}}}{H_{\text{dl}}}
\]
Modulation Transfer Function

Area under the MTF
Retinal Sampling
Sampling by Foveal Cones

Projected Image

Sampled Image

20/20 letter

5 arc minutes
Sampling by Foveal Cones

Projected Image

Sampled Image

20/5 letter

5 arc minutes
Nyquist Sampling Theorem
Photoreceptor Sampling >> Spatial Frequency

nearly 100% transmitted
Photoreceptor Sampling = 2 x Spatial Frequency

nearly 100% transmitted
Photoreceptor Sampling = Spatial Frequency

nothing transmitted
Photoreceptor Sampling < Spatial Frequency
Nyquist theorem:
The maximum spatial frequency that can be detected is equal to $\frac{1}{2}$ of the sampling frequency.

foveal cone spacing ~ 120 samples/deg

maximum spatial frequency: 60 cycles/deg (20/10 or 6/3 acuity)
MTF: Cutoff Frequency

Nyquist limit

modulation transfer

1

0.5

0

spatial frequency (c/deg)

0 50 100 150 200 250 300

1 mm

2 mm

4 mm

6 mm

8 mm

\[ f_{\text{cutoff}} = \frac{a}{57.3 \cdot \lambda} \]

Rule of thumb: cutoff frequency increases by ~30 c/d for each mm increase in pupil size

cut-off frequency